29th Conference of the Middle European Cooperation in Statistical Physics,

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Sine-Gordon model in finite volume

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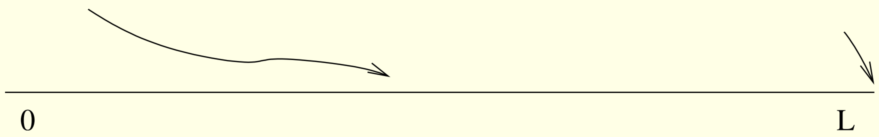

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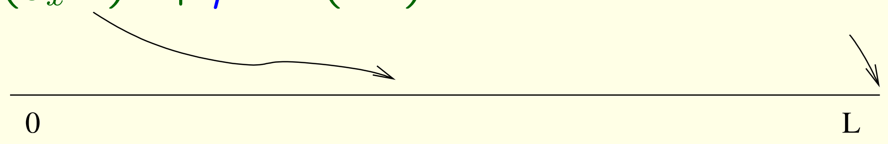

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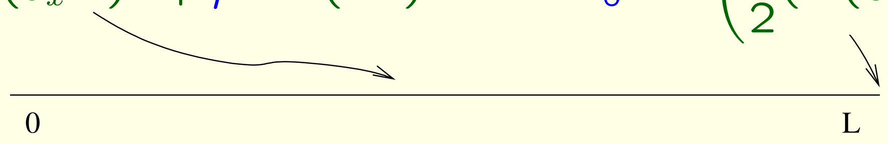

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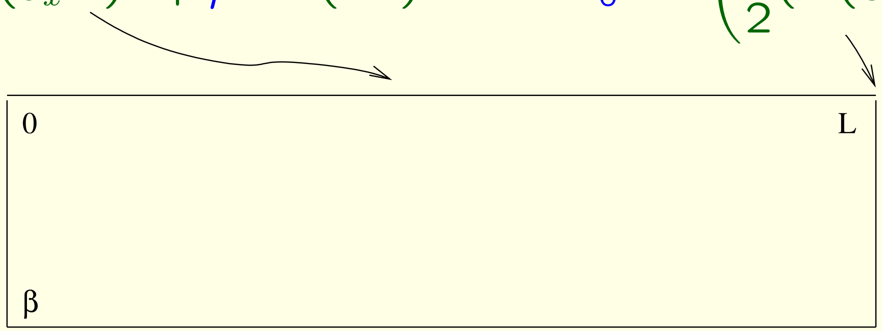

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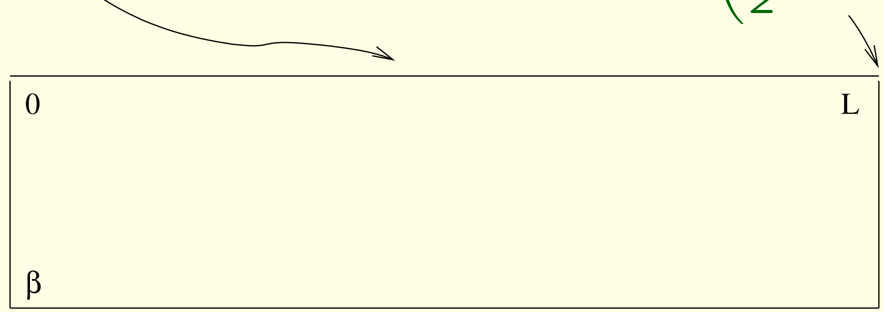

Partition function for large $\beta \leftrightarrow$ ground state energy $E_0(L)$

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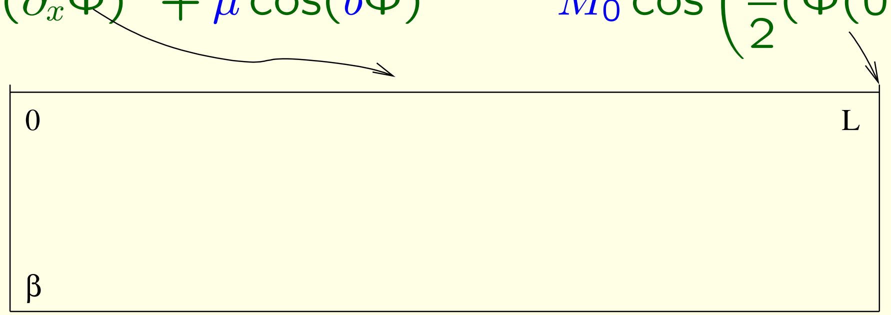

Partition function for large $\beta \leftrightarrow$ ground state energy $E_0(L)$

$$\boxed{Z(\beta,L)\propto e^{-\beta E_0(L)}}$$

Plan of the talk

Plan of the talk

Periodic

Plan of the talk

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Why do we need $E_0(L)$?

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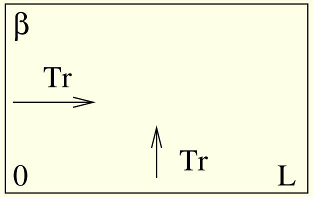

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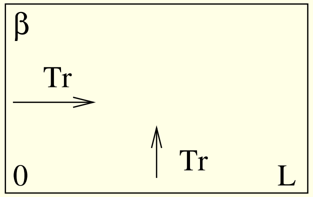

exact $L = \infty$, Scattering

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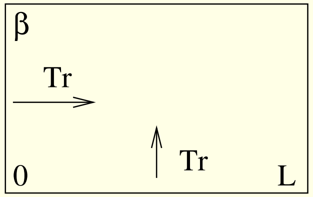

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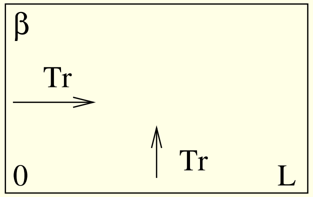

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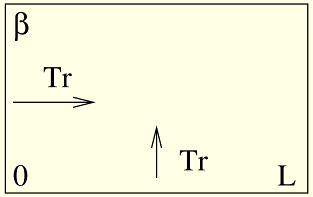

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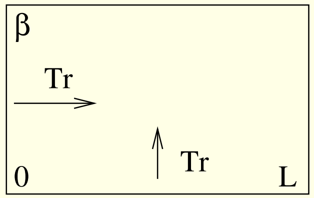

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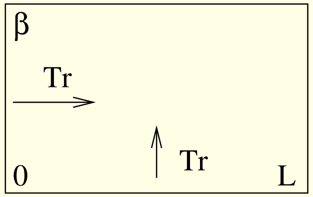

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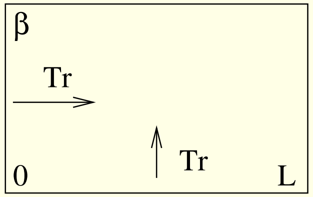

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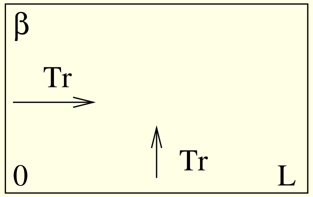

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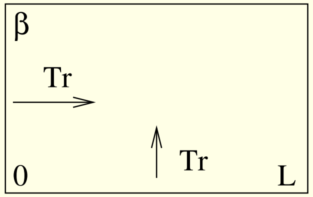

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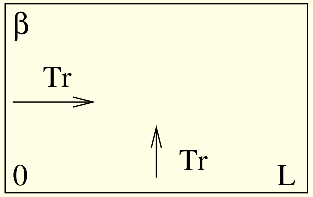

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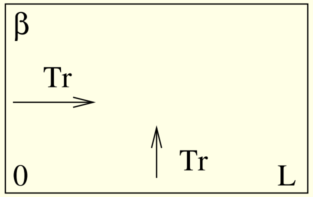

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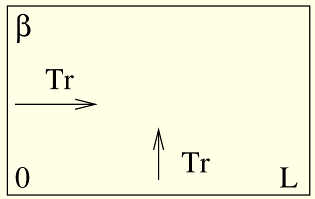

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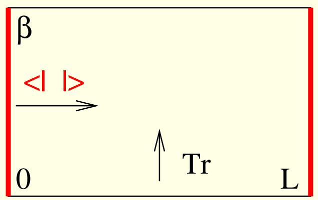

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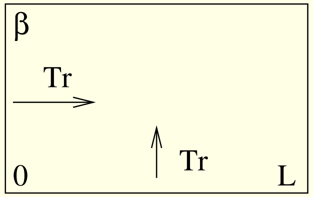

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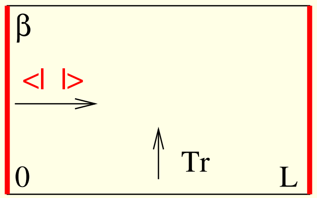

Exact $L \rightarrow \infty$, Reflection

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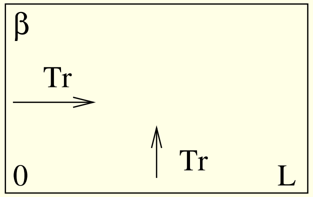

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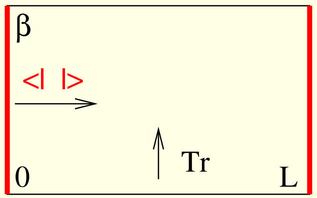

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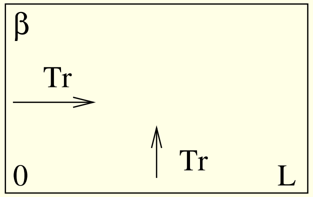

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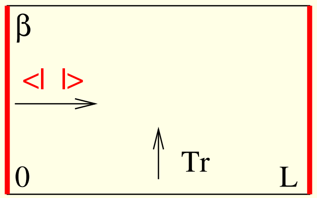

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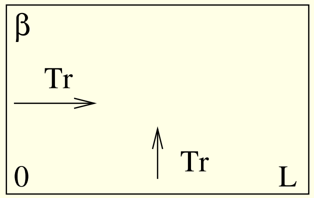

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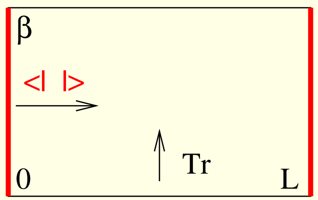

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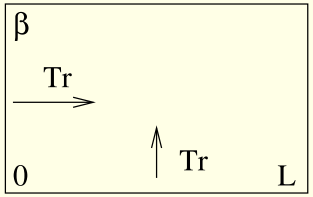

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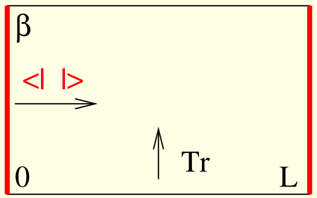

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Motivation: bulk

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Z.B., L. Palla, G. Takacs, F. Wagner

Nucl. Phys. B 587 (2000) 585.

Numerical spectrum of $H = \ldots + \mu \cos(b\Phi)$:

$\frac{H}{M}$ plotted against ML

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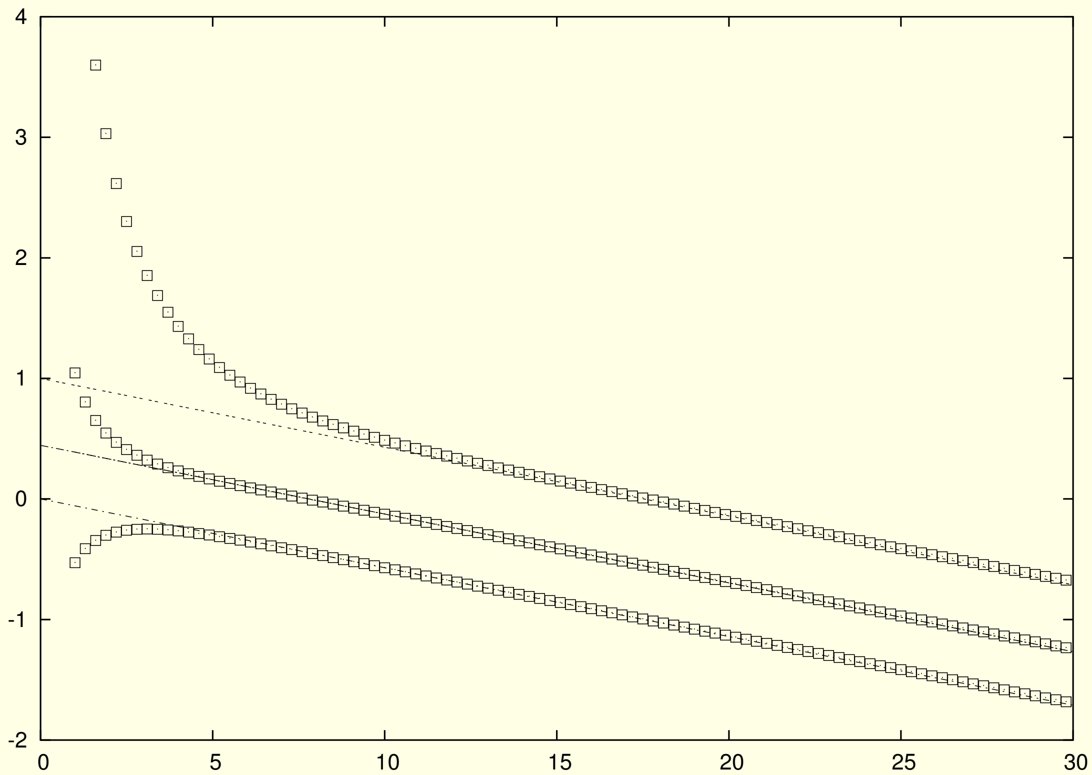

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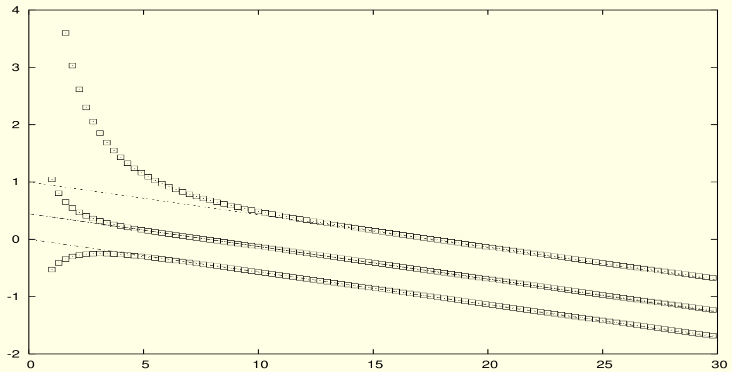

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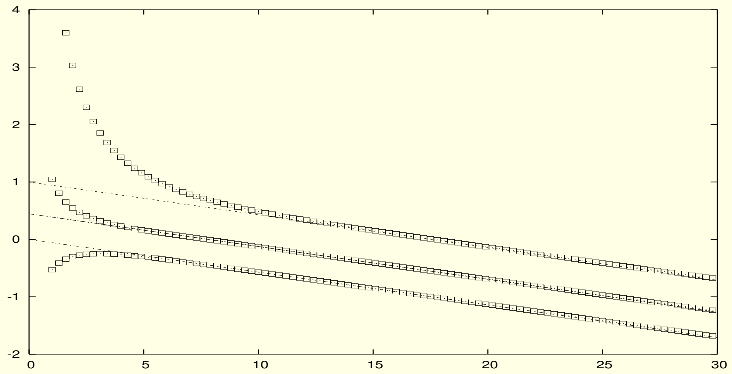

Ground state $E_o(L) = M^2 \epsilon_{bulk}(b) L + \ldots$ Gaps $m(b), M(b)$

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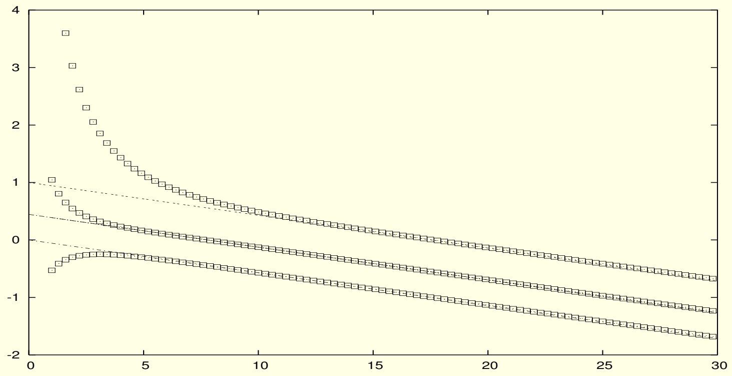

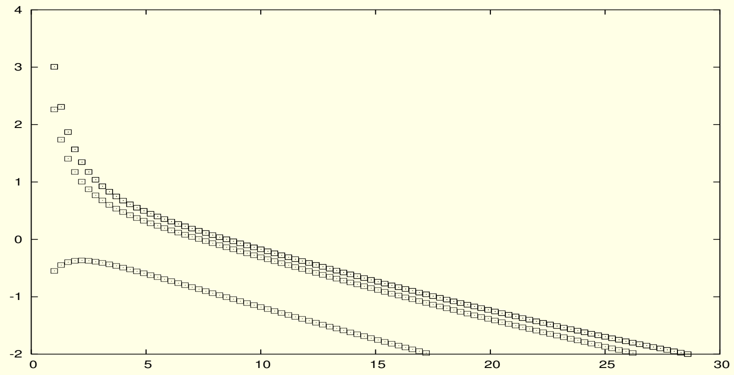

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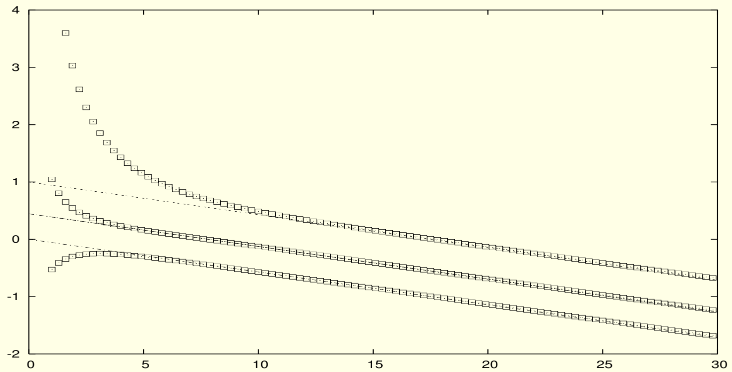

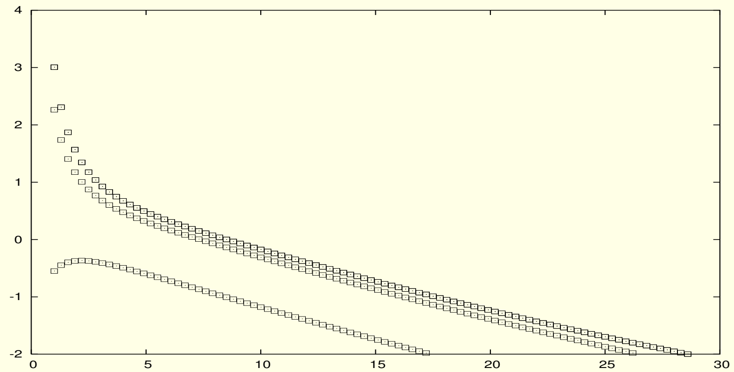

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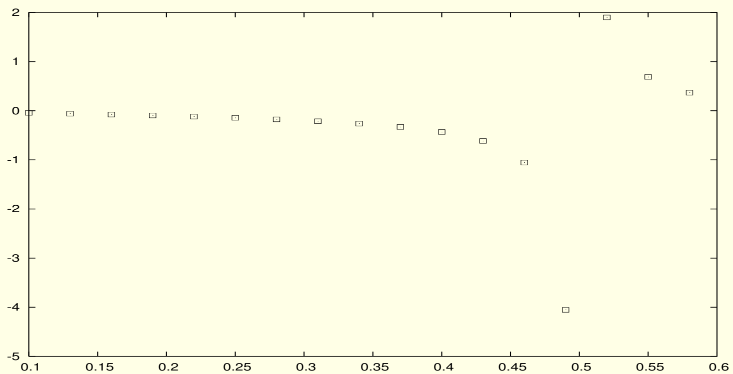

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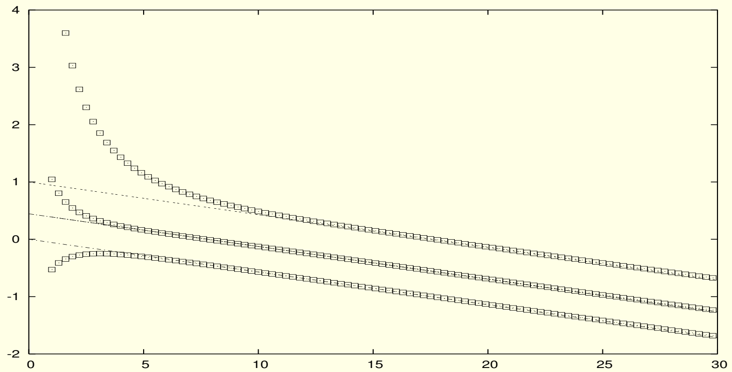

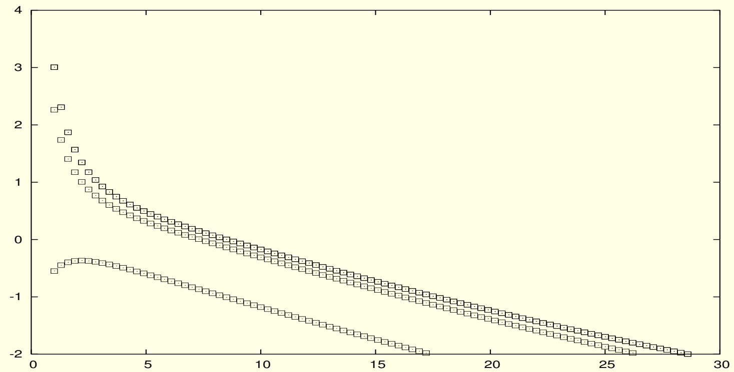

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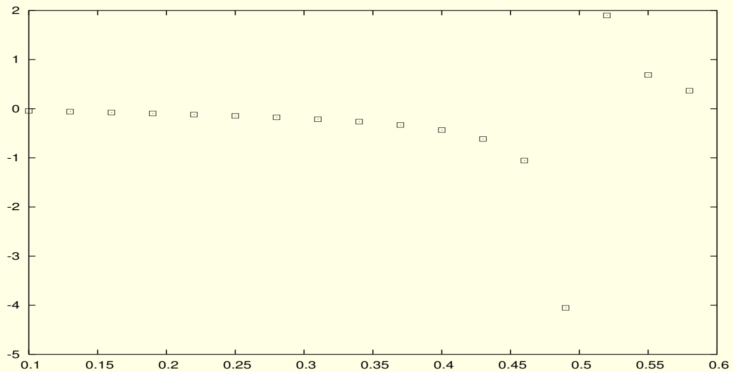
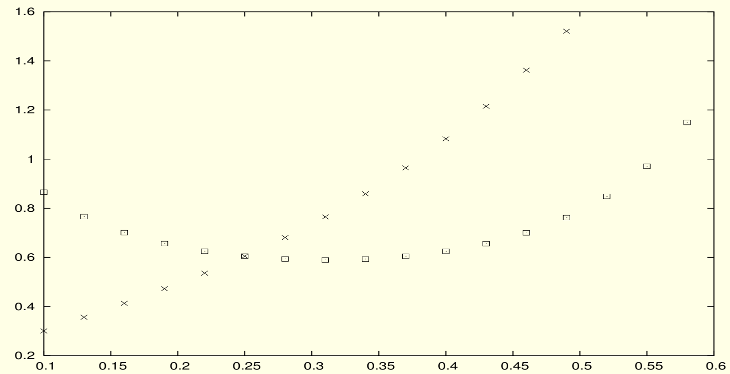
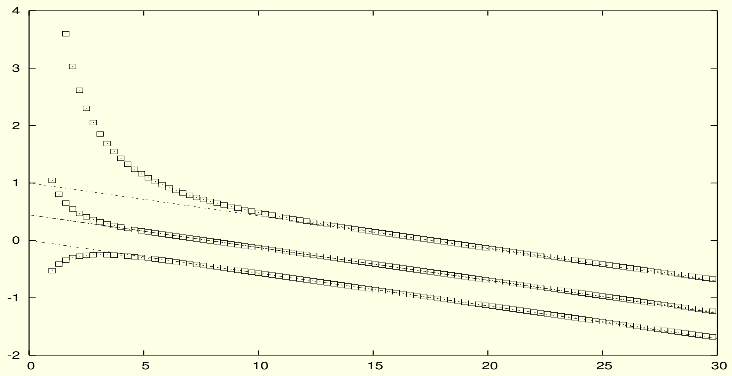


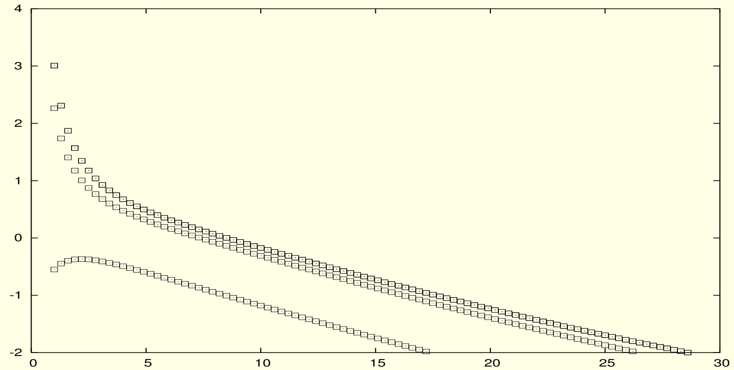


Motivation: bulk

Z.B., L. Palla, G. Takacs,
F. Wagner
Nucl. Phys. B 587 (2000) 585.

Numerical spectrum of $H = \ldots + \mu\cos(b\Phi)$:
$\frac{H}{M}$ plotted against ML





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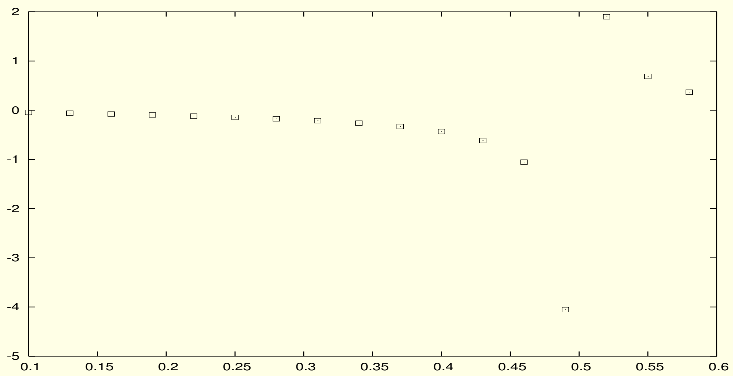

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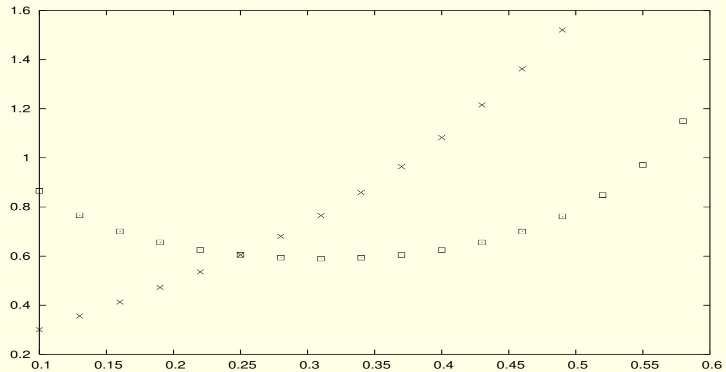

Variation as a function of $\frac{b^2}{8\pi}$

Idea for spin models

Idea for spin models

Isotropic spin model on a square, periodic lattice

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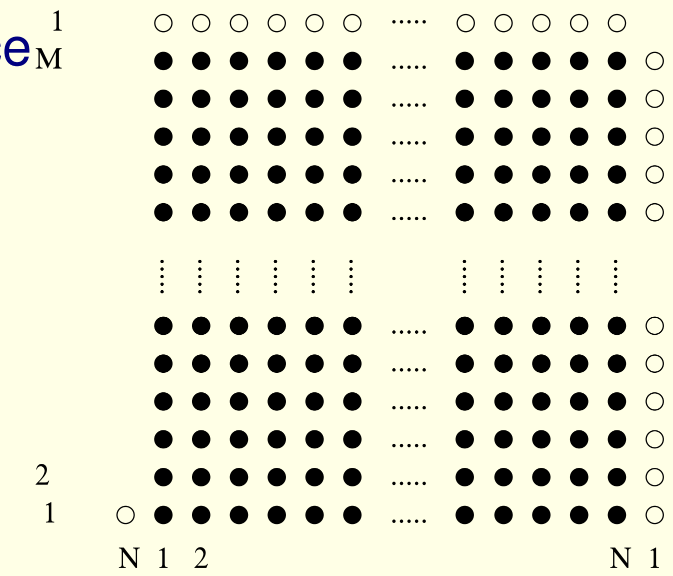

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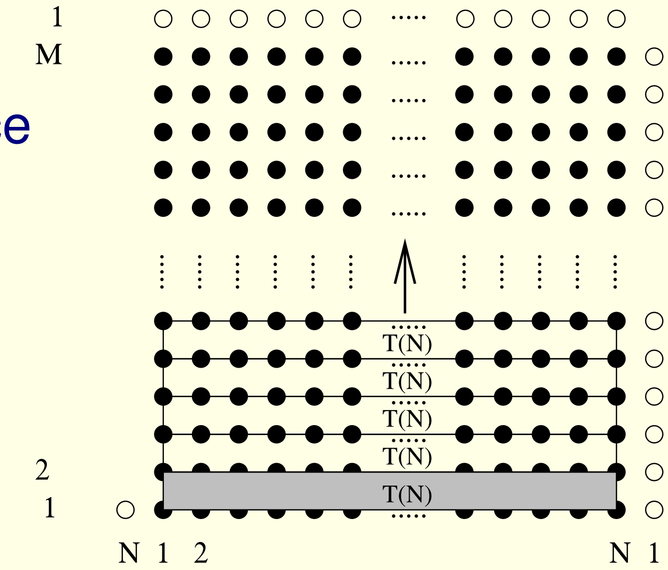

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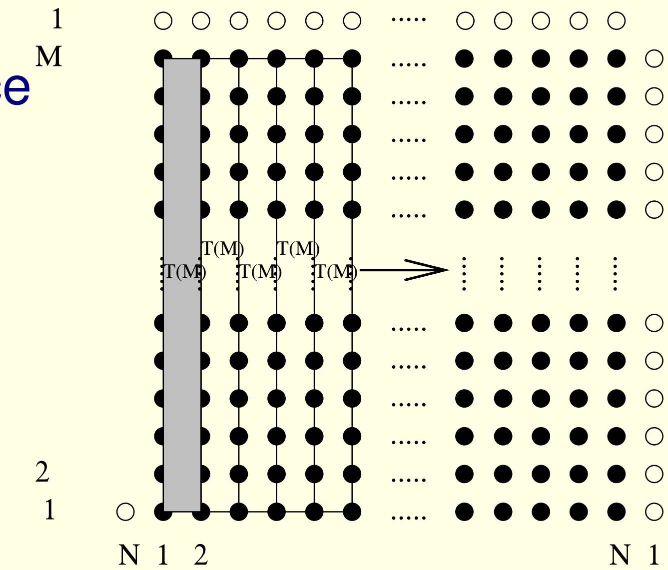

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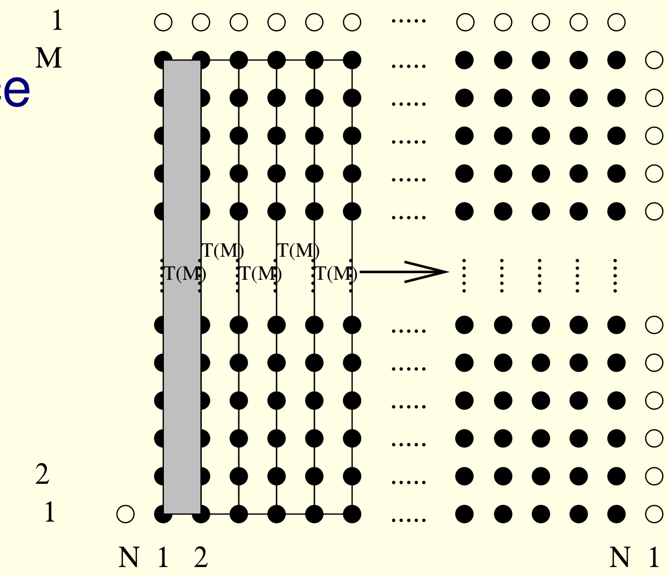

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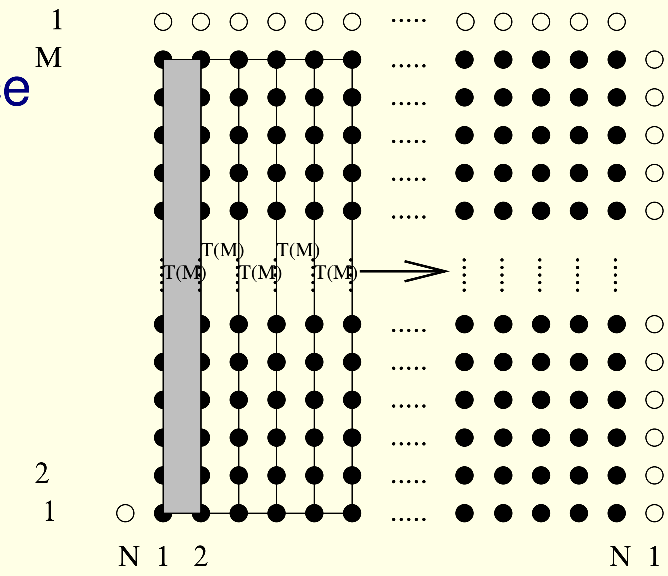

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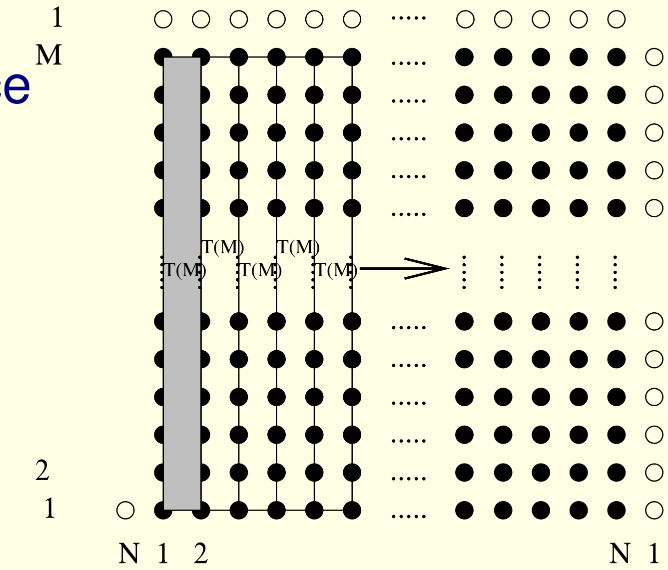

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Field theory on a domain of size (L, β)

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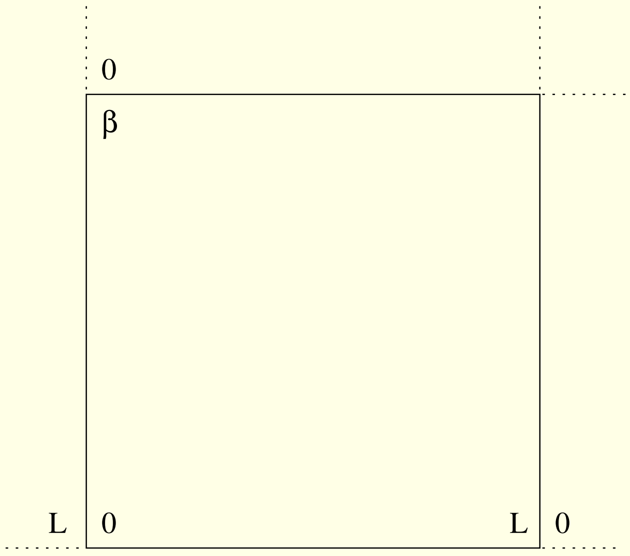

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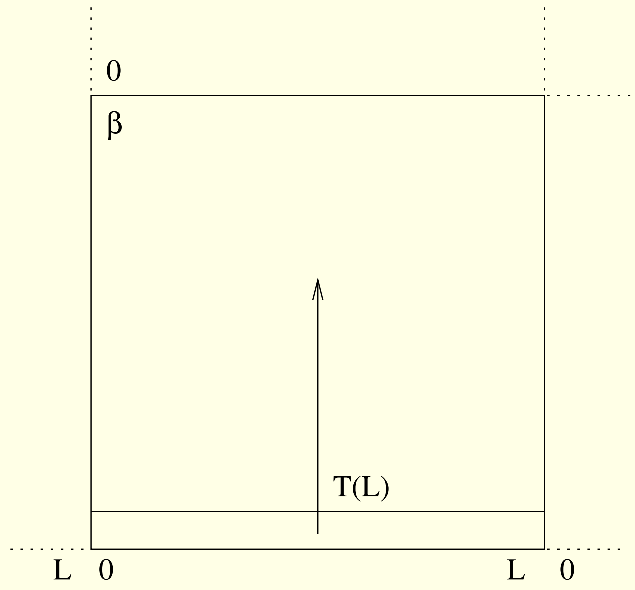

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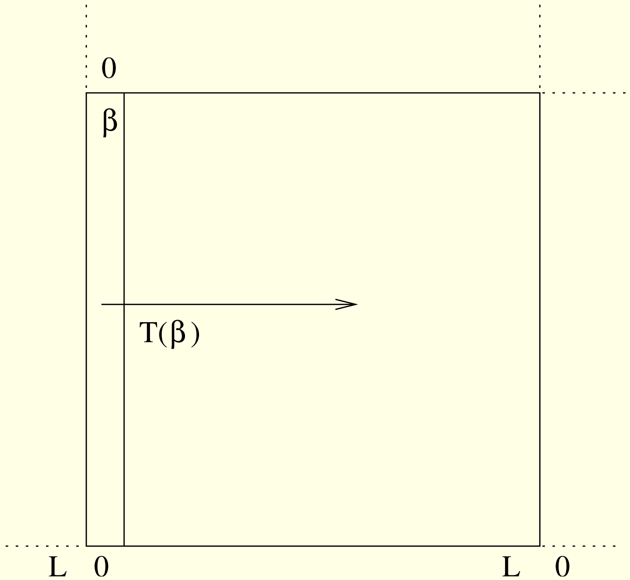

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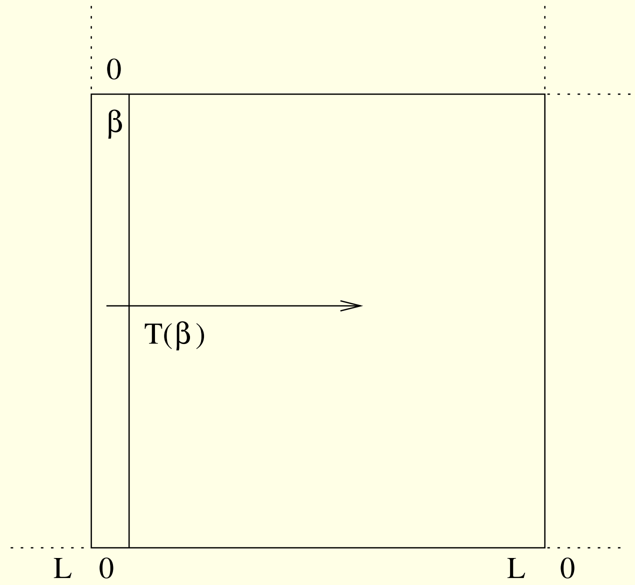

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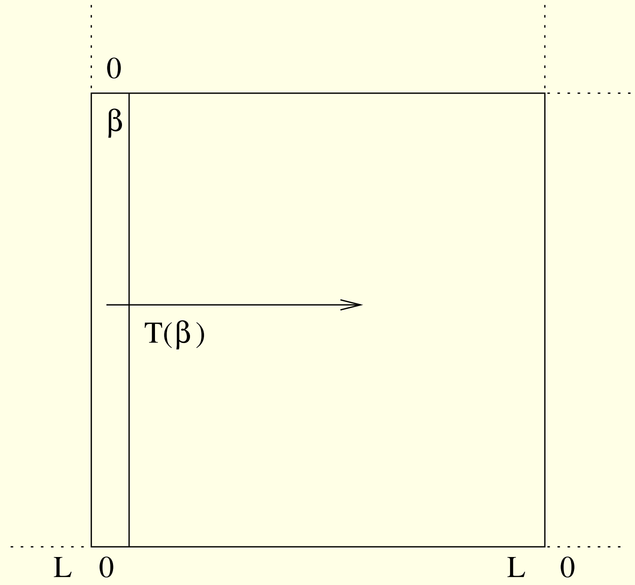

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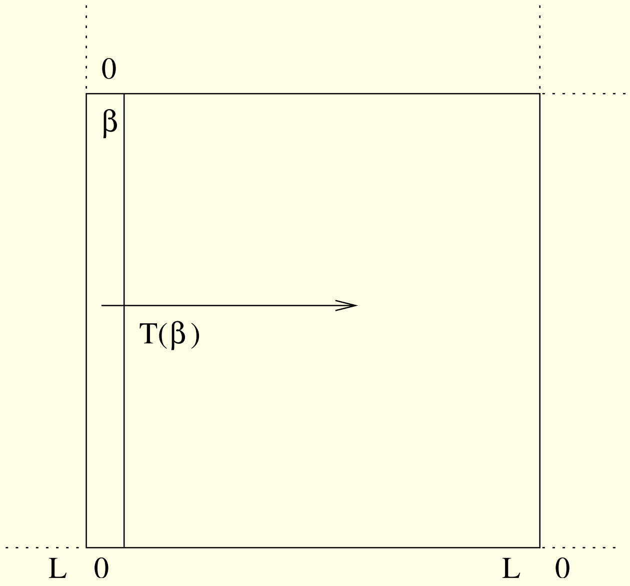

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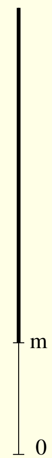

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Particles in finite volume: momentum quantization

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'Very' large volume spectrum $O(e^{-L})$

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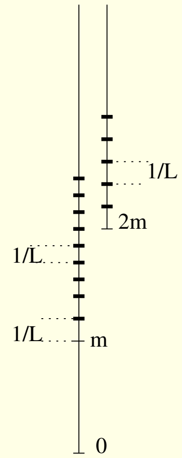

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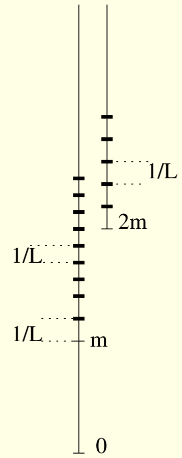

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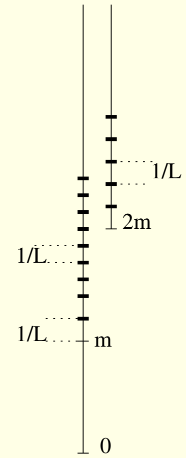

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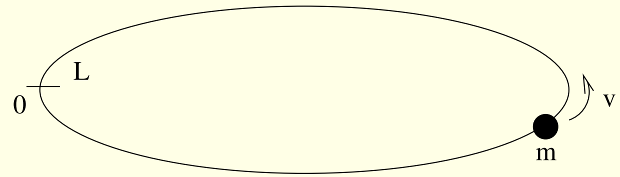

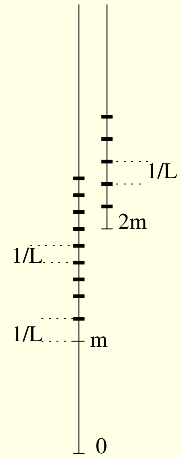

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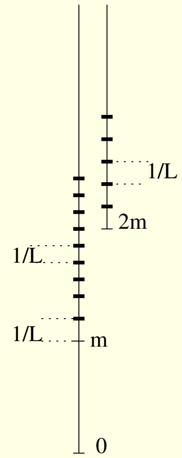

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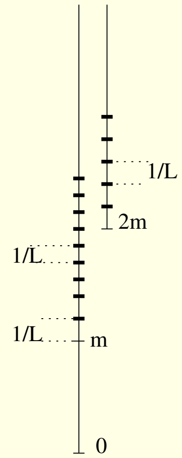

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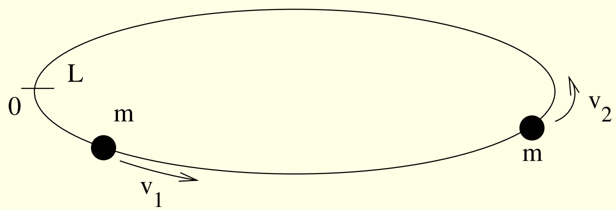

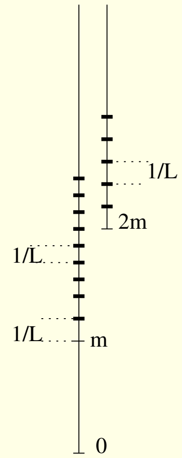

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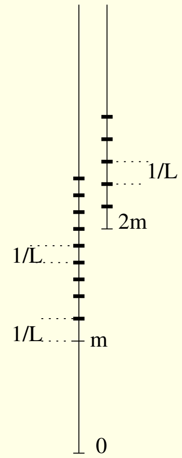

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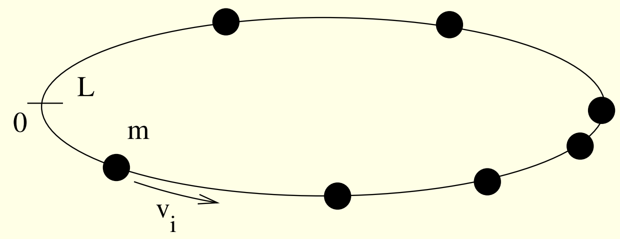

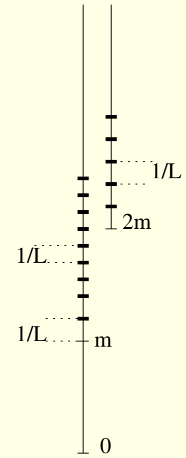

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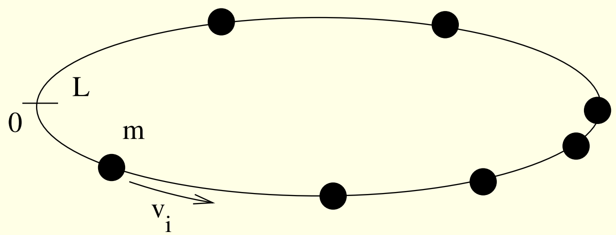

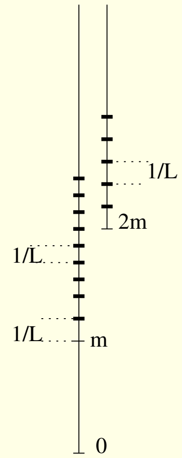

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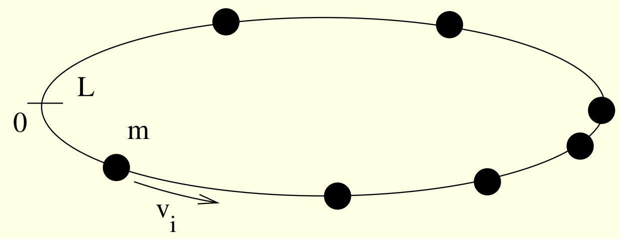

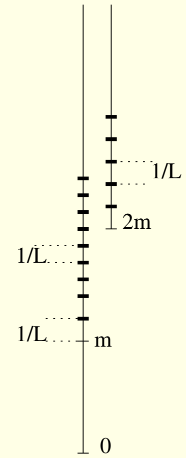

One particle states

Momentum quantization: $e^{im \sinh \theta L} = 1 \quad \to E = m \cosh \theta(n, L)$

Two particle states

Momentum quantization $\quad e^{im \sinh \theta_1 L} e^{i\delta(\theta_1 - \theta_2)} = 1$

Multiparticle states

Momentum quantization $m \sinh(\theta_i) L + \sum_j \delta(\theta_i - \theta_j) = 2\pi n_i$

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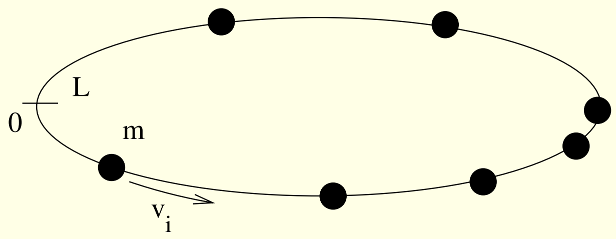

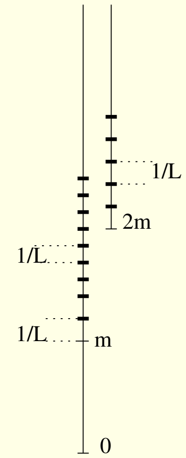

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Partition function $Z(L,\beta) = \sum_{\{\theta_i\}} e^{-\beta m \sum_i \cosh\theta_i}$

Thermodynamic Bethe Ansatz (TBA)

Al. B. Zamolodchikov:
hep-th/0005181

Thermodynamic Bethe Ansatz (TBA)

Dominant contribution: finite particle density states

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Introduce particle and hole densities: ρ, ρ_h

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Saddle point particle density

$$\epsilon(\theta) = m\beta\cosh(\theta) + \int \delta'(\theta-\theta')(1+e^{-\epsilon(\theta')})d\theta' \quad \epsilon(\theta) = -\ln\frac{\rho(\theta)}{\rho_h(\theta)}$$

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Ground state energy exactly:

$$\boxed{LE_0(L) = \int \cosh(\theta)(1 + e^{-\epsilon(\theta)}) d\theta}$$

A. B. Zamolodchikov,
Al. B. Zamolodchikov:
Ann. Phys. 120 (1979) 253.

Sine-Gordon model

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sine-Gordon TBA

R. Tateo

Phys. Lett. B355 (1995) 157.

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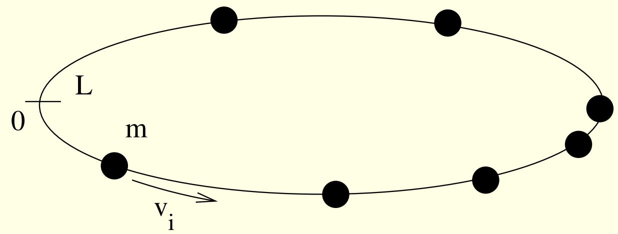

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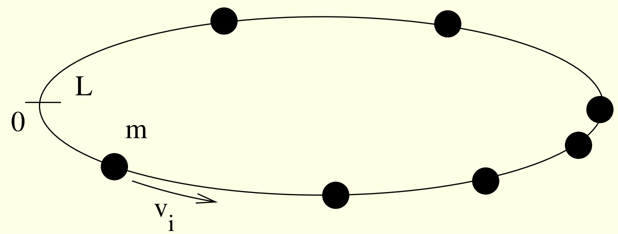

Rapidities are determined by

$e^{ip_i L} \prod_j S_{ij}(\theta_i - \theta_j) = 1 \rightarrow$ Diagonalization: XXZ

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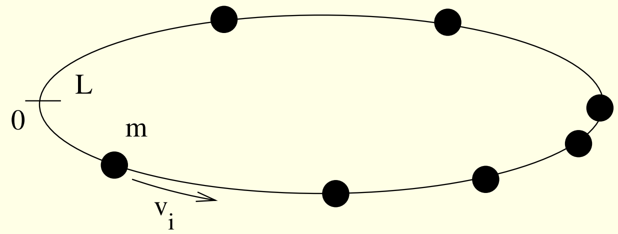

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sine-Gordon TBA

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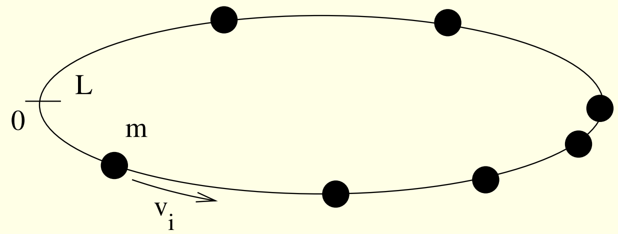

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Coupled real integral equations

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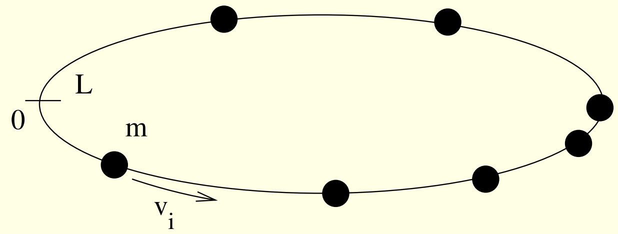

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By summing up: one complex integral equations

C. Destri, H. de Vega
Nucl. Phys. B 358 (1991) 251.

Conclusion: bulk

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exact $E_0(L, \lambda, m)$

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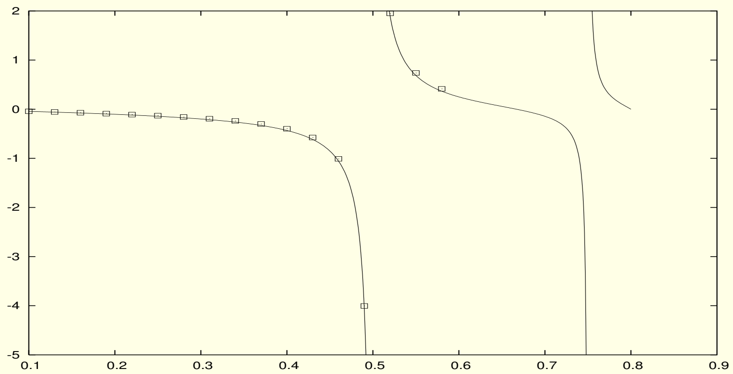

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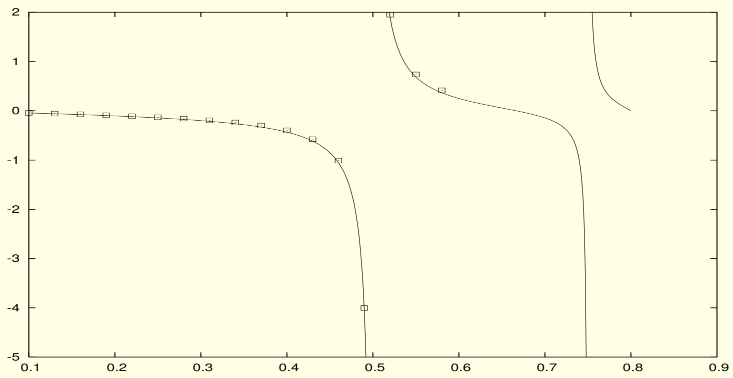

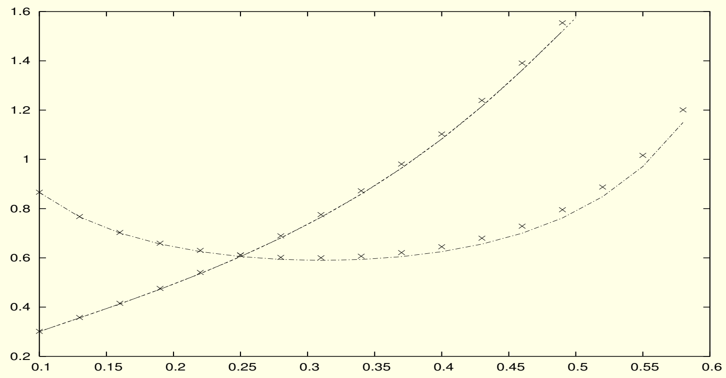

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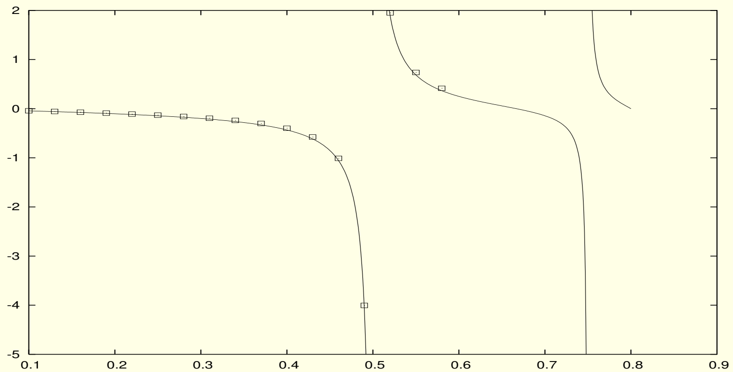
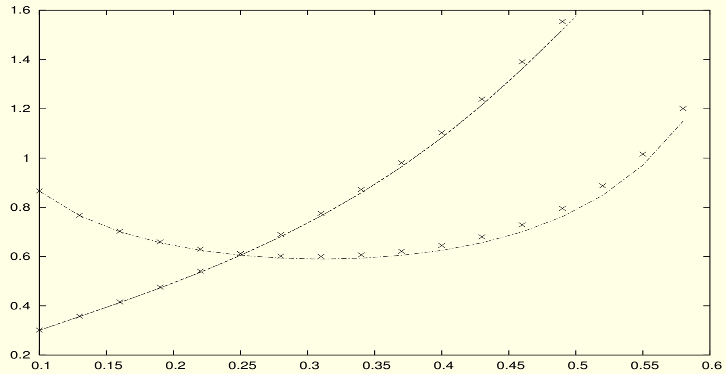
Int.J.Mod.Phys. A 10 (1995)1125.

$$\lambda = \frac{8\pi}{b^2} - 1 \quad ; \qquad M(b,\mu) = \frac{2\Gamma(\frac{b^2}{16\pi - b^2})}{\sqrt{\pi}\Gamma(\frac{4\pi}{8\pi - b^2})}\left(\mu \frac{\pi\Gamma(1-\frac{b^2}{8\pi})}{2\Gamma(\frac{b^2}{8\pi})}\right)^{\frac{4\pi}{8\pi - b^2}}$$

Bulk energy $\epsilon_{bulk}(\lambda) = -\frac{1}{4}\tan(\frac{\pi}{2\lambda})$ Gaps: $m = 2M\sin(\frac{\pi}{2\lambda}),\ M$





Variation as a function of $\frac{b^2}{8\pi}$

Motivation: boundary

Motivation: boundary

Z.B., L. Palla, G. Takacs,
Nucl. Phys. B 622 (2002) 565.

Numerical spectrum of $\ldots + M_0 \cosh\left(\frac{b}{2}(\Phi(0) - \varphi_0)\right)$:

$\frac{H^B}{M}$ plotted against ML

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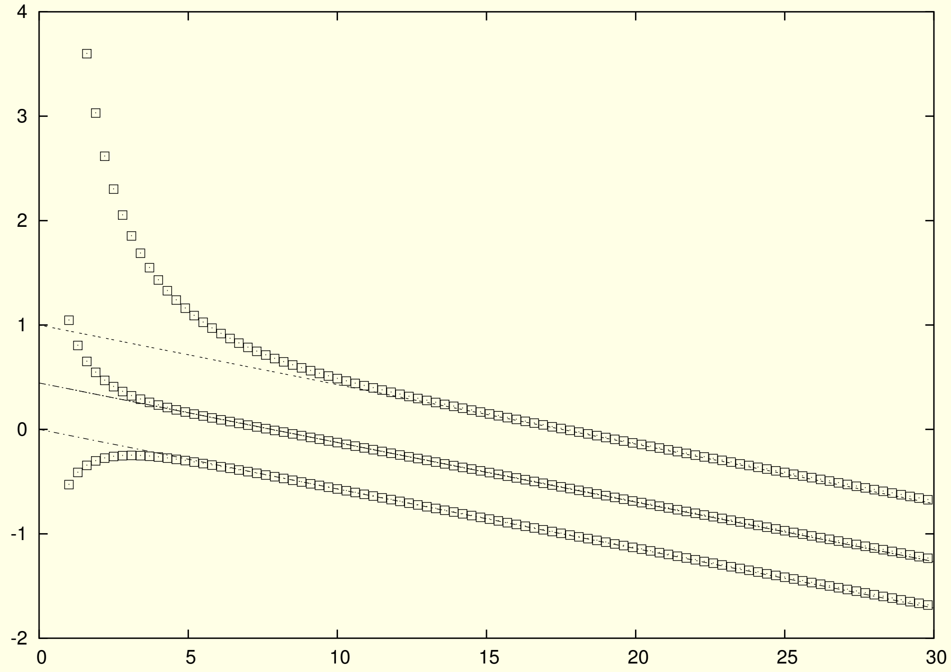

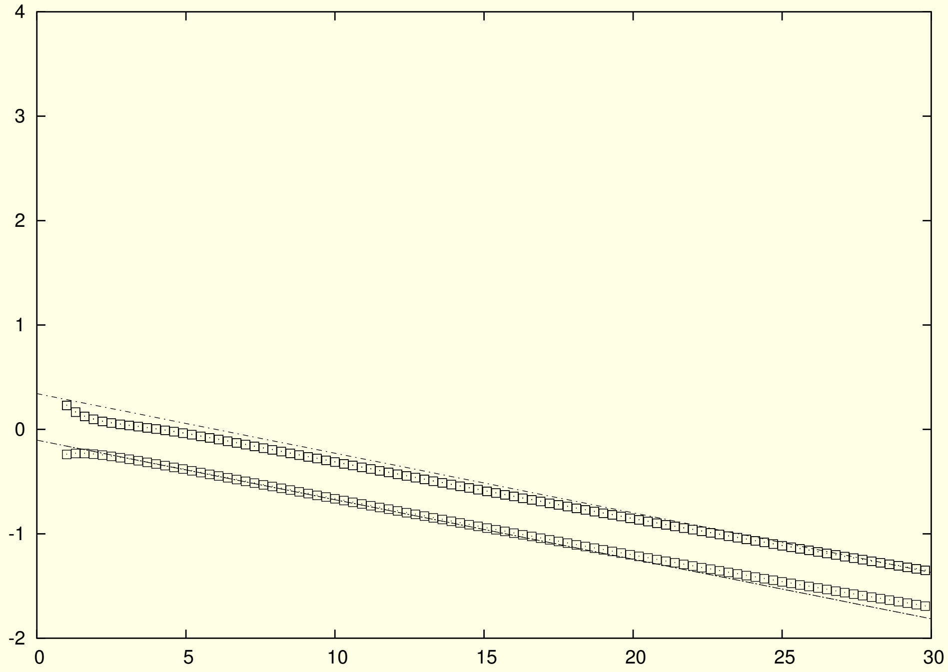

Bulk

Boundary

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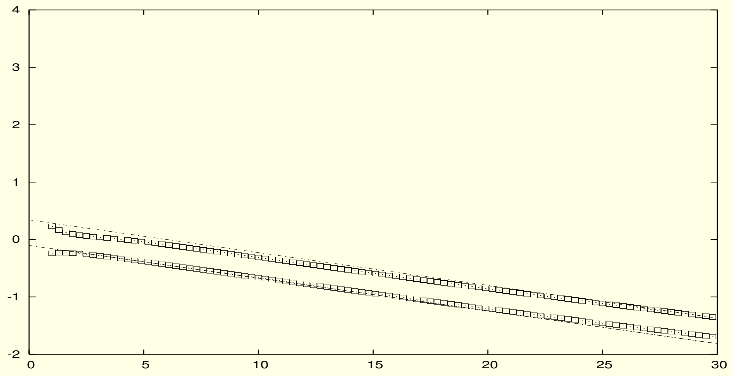

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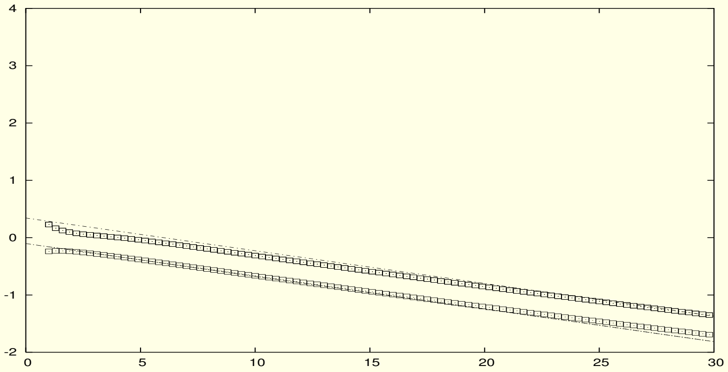

Ground state $E_o(L) = M^2(\ldots + \epsilon_{bound}(M_0, \varphi_0)) + \ldots$ Gap $m_{bound}(M_0, \varphi_0)$

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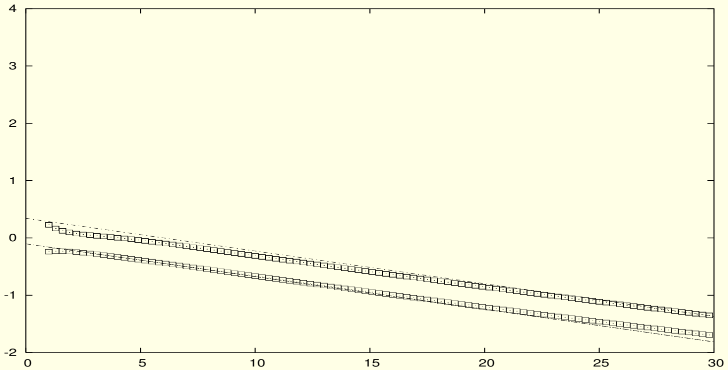

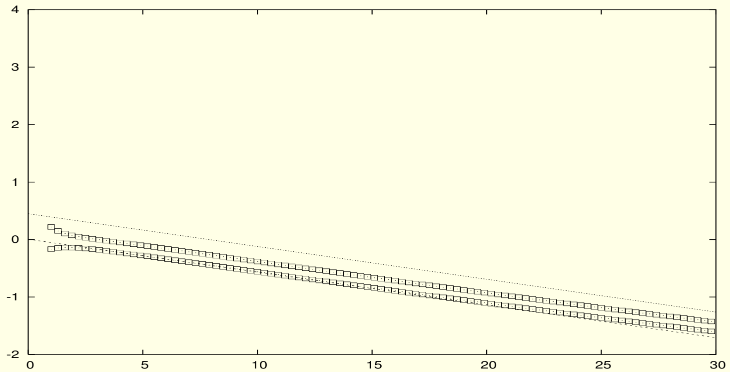

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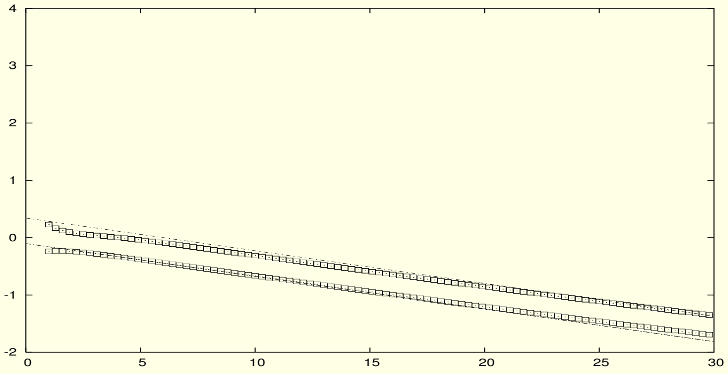

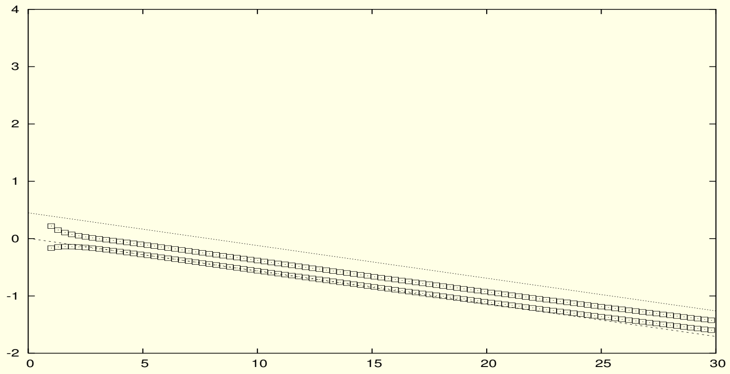

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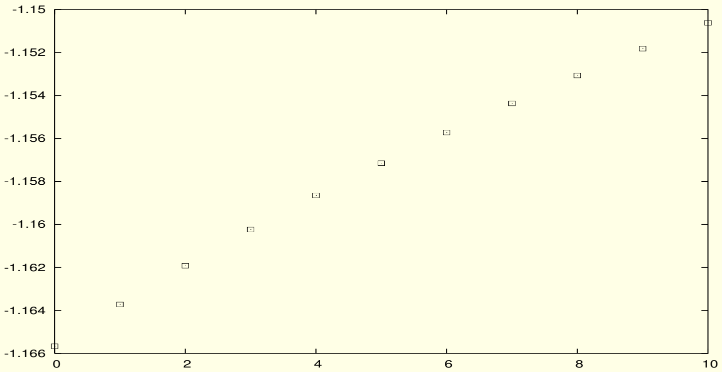

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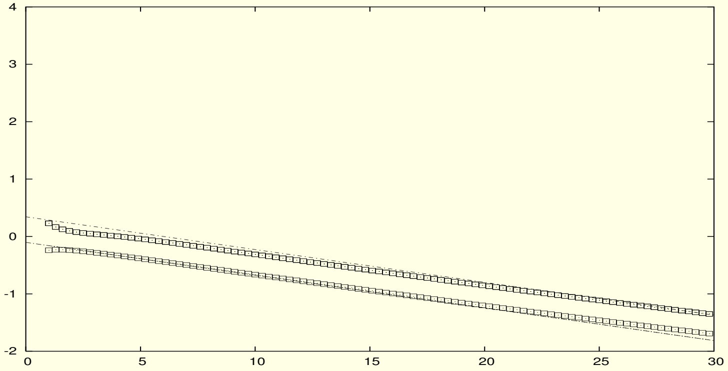

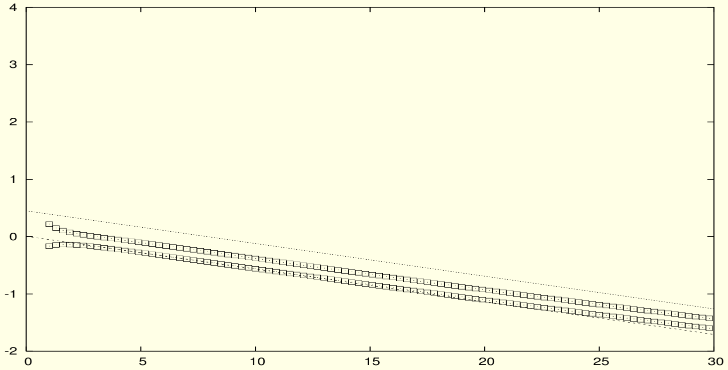

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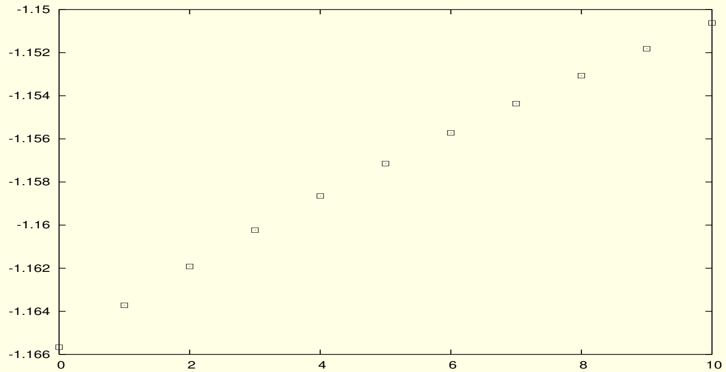

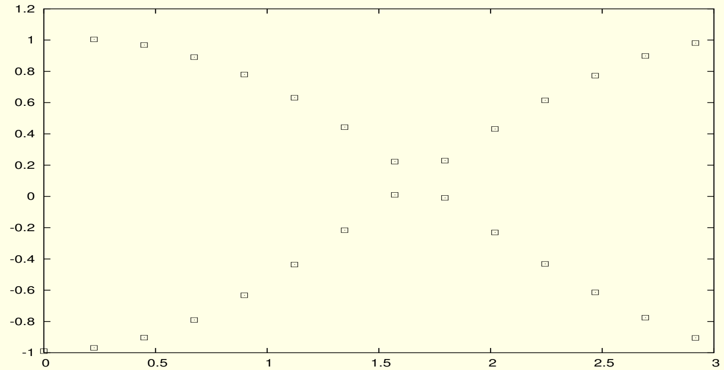

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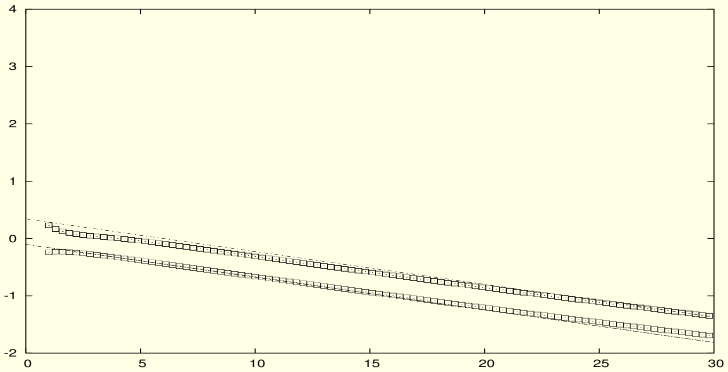

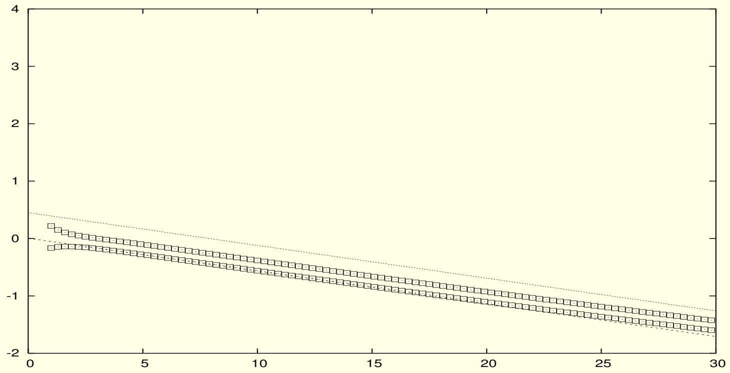

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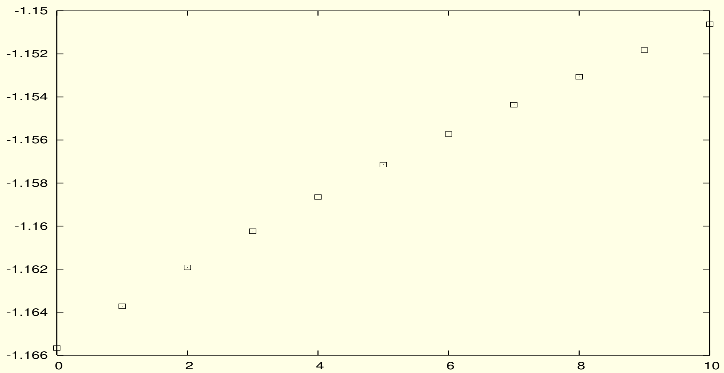
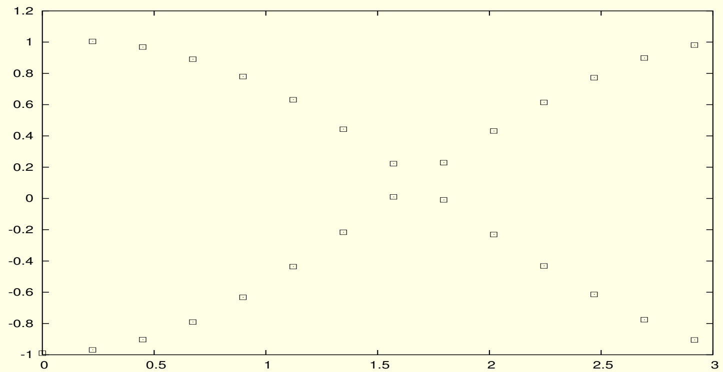




Variation as a function of $M_0 = 0, 10$ $\qquad \frac{b\varphi_0}{2} = 0, \pi$

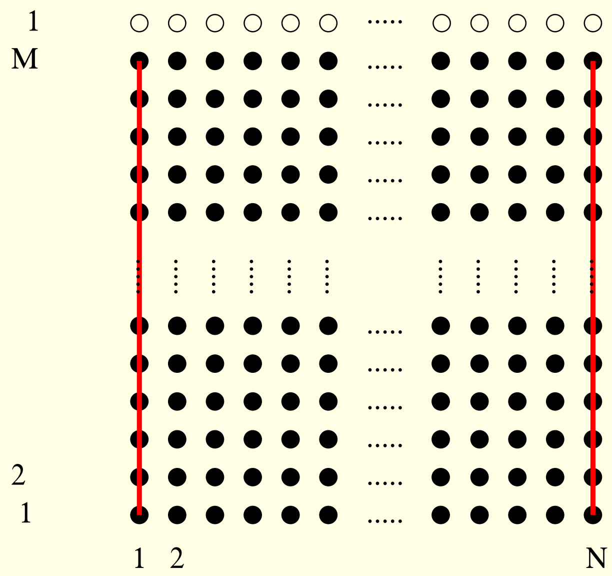
Idea for spin models

Idea for spin models

Isotropic spin model on a cylindrical lattice

Idea for spin models

Isotropic spin model on a cylindrical lattice



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Hamiltonian: $H^B(N) = \sum_{i=1}^N h_i + b_1 + b_N$

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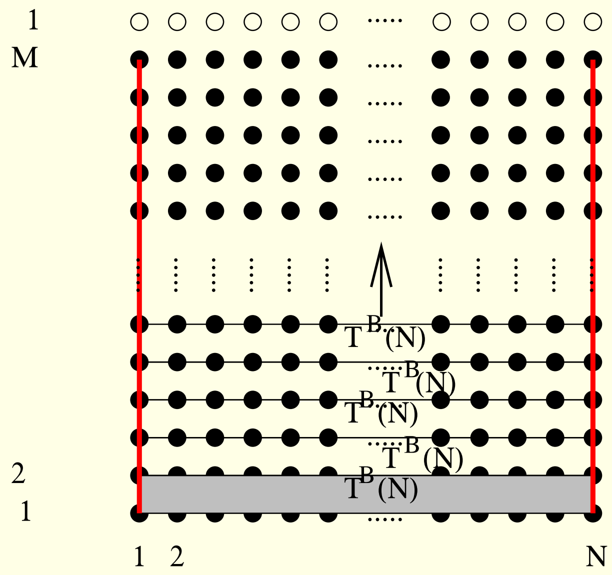
Idea for spin models



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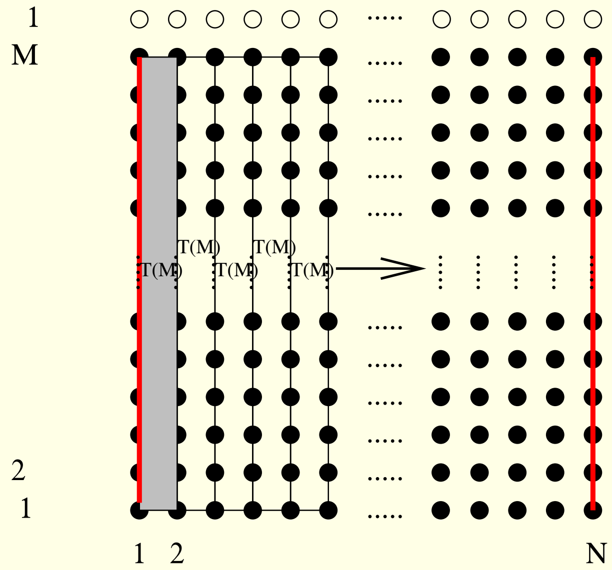

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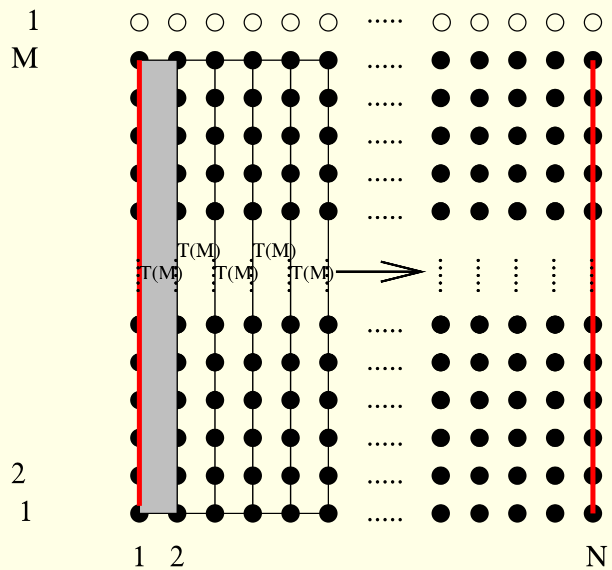

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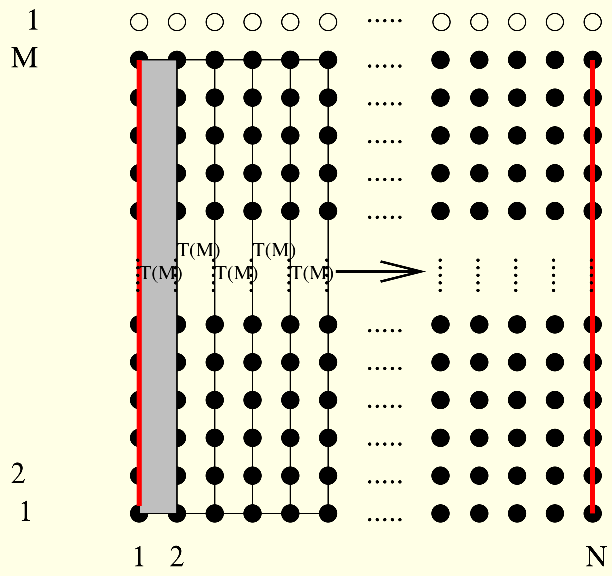

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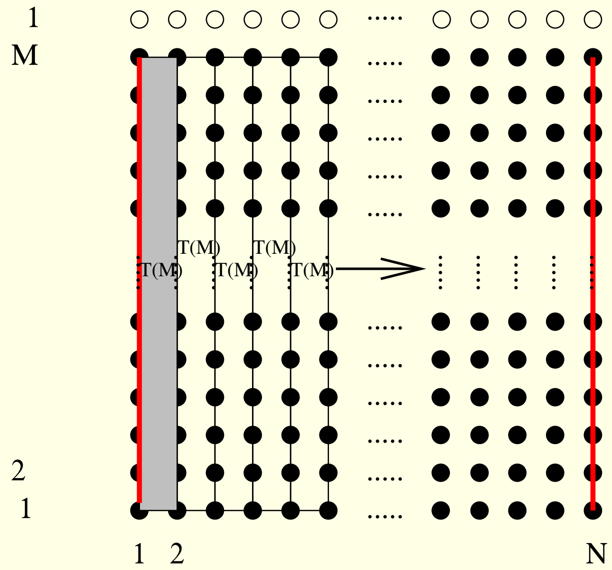

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Adaptation for field theories

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Field theory on the cylinder of size (L, β)

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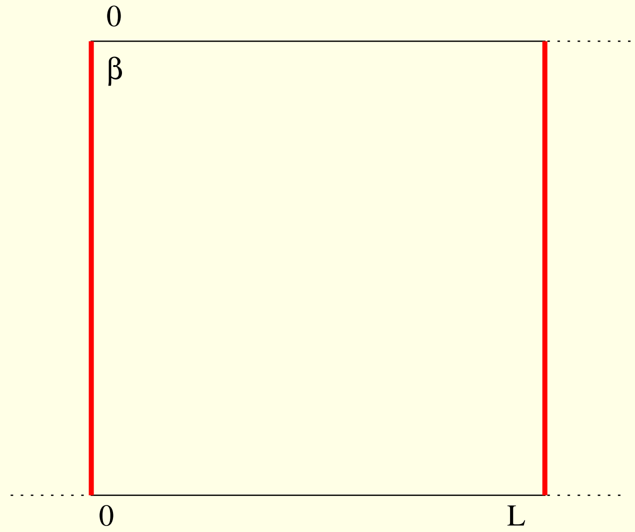

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Hamiltonian: $H^B(L) = \int_0^L h(x)dx + b(0) + b(L)$

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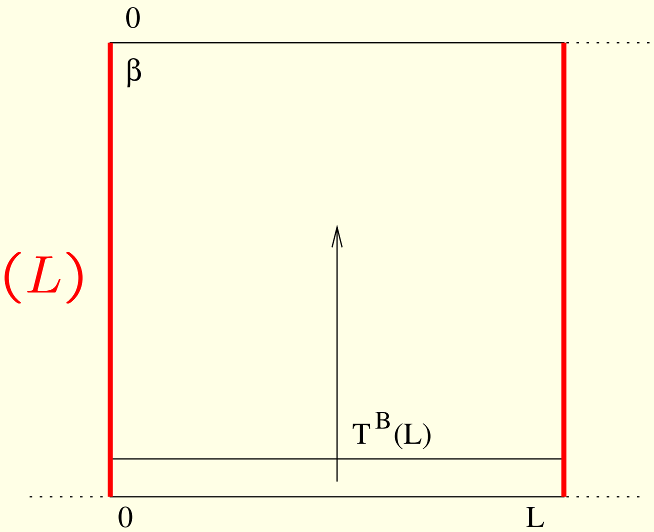

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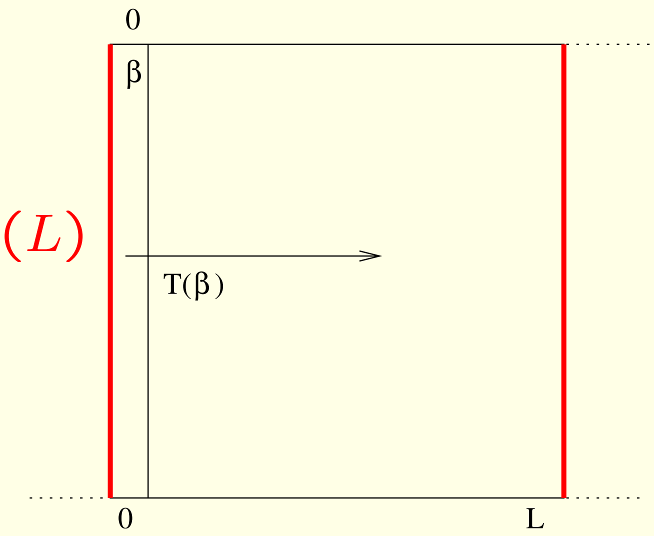

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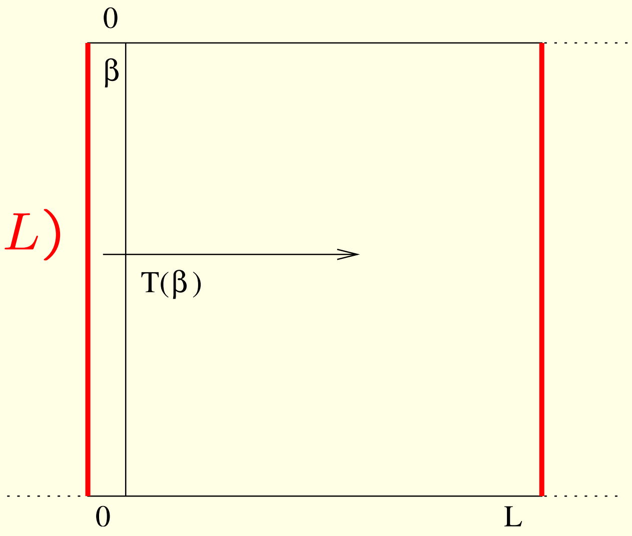

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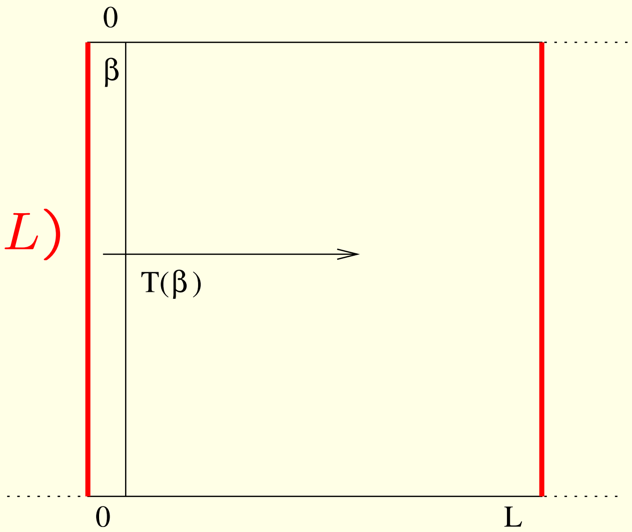

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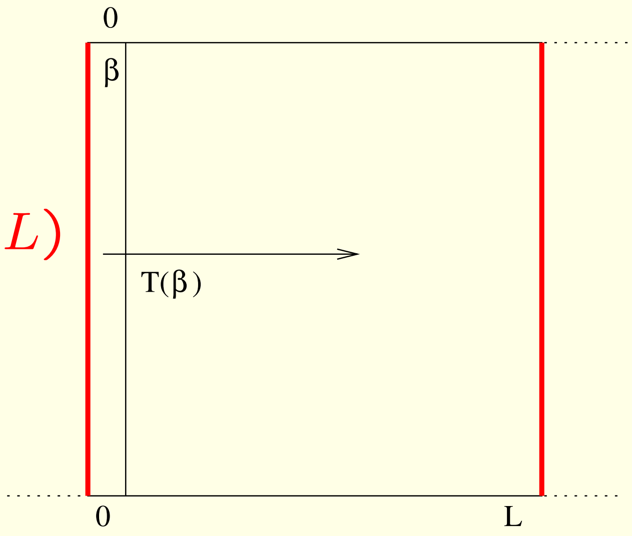

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Ground state energy for all L, $Z(L,\beta) \to e^{-E_0^B(L)\beta} + \dots \quad \to E_o^B(L)$

Boundary sinh-Gordon model

+

Boundary sinh-Gordon model

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Nucl. Phys. B 622 (2002) 565.

Hamiltonian

+

Boundary sinh-Gordon model

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Reflection $B(\theta) \to B(-\theta)$ with amplitude $R(\theta, \eta, \Theta) = e^{i\delta^R(\theta)}$

Matrix elements $\langle b_0 | 2n \rangle \langle 2n | b_L \rangle = \prod_i^n \bar{R}_0(\frac{i\pi}{2} - \theta_i) R_L(\frac{i\pi}{2} - \theta_i)$

Multiparticle eigenstates with opposite rapidities :

$\{\theta_i\} = \{\theta_1, -\theta_1, \theta_2, -\theta_2, \ldots, \theta_n, -\theta_n\}$

Boundary sinh-Gordon model

Z.B., L. Palla, G. Takacs, Nucl. Phys. B 622 (2002) 565.

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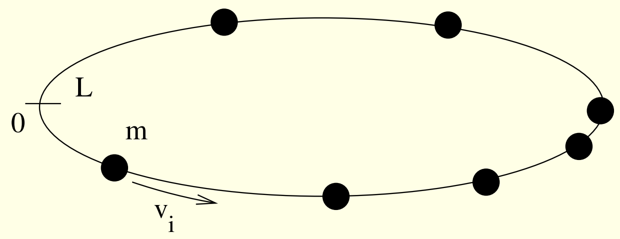

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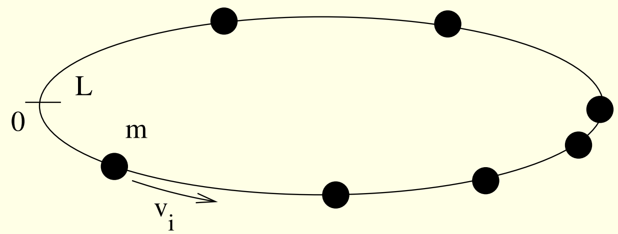

Rapidities are determined by

$$m \sinh(\theta_i) L + \delta(2\theta_i) + \sum [\delta(\theta_i - \theta_j) + \delta(\theta_i + \theta_j)] = 2\pi n_i$$

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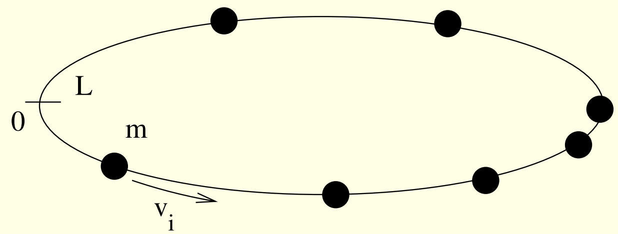

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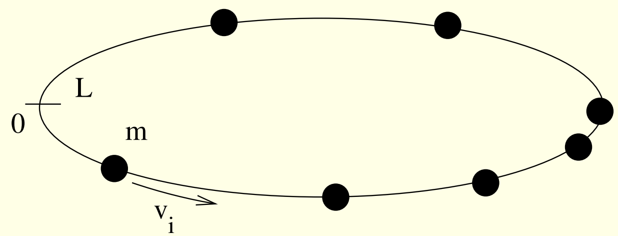

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Energy: $E = 2 \sum_{j=1}^n m \cosh \theta_i$

Partition function $Z(L, \beta) = \sum_{\{\theta_i\}} e^{-\beta \sum_i \left(2m \cosh \theta_i + \log \bar{R}_0(\frac{i\pi}{2} - \theta_i) R_L(\frac{i\pi}{2} - \theta_i) \right)}$

Boundary TBA

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Saddle point particle density in terms of $\epsilon(\theta) = -\ln \frac{\rho(\theta)}{\rho_h(\theta)}$

$$\epsilon(\theta) = 2mL\cosh(\theta) + \int [\delta'(\theta - \theta') + \delta'(\theta + \theta')](1 + e^{-\epsilon(\theta')})d\theta'$$

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Ground state energy exactly:

$$LE_0(L) = \int \cosh(\theta)(1 + 2\bar{R}_0(\tfrac{i\pi}{2} - \theta)R_L(\tfrac{i\pi}{2} - \theta)e^{-\epsilon(\theta)})d\theta$$

Boundary sine-Gordon model

+

Boundary sine-Gordon model

Hamiltonian

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Boundary sine-Gordon model

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Reflection of soliton doublet

S. Ghoshal., A.B. Zamolodchikov
Int. J. Mod. Phys. A 9 (1994) 3841.

$$\begin{pmatrix} \cos(i\lambda\theta)\cos\eta\cosh\Theta + (\cos\leftrightarrow\sin) & \cos i\lambda\theta \sin i\lambda\theta \\ \cos i\lambda\theta \sin i\lambda\theta & \cos(i\lambda\theta)\cos\eta\cosh\Theta - (\cos\leftrightarrow\sin) \end{pmatrix}$$

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work in progress

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Analitically continue the sinh-Gordon results

Conclusion: Boundary

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$$\epsilon_{bound}(\lambda)=-\frac{1}{2\cos(\pi/2\lambda)}\left(\cos(\tfrac{\eta}{\lambda})+\cosh(\tfrac{\Theta}{\lambda})-\tfrac{1}{2}\cos(\tfrac{\pi}{2\lambda})+\tfrac{1}{2}\sin(\tfrac{\pi}{2\lambda})-\tfrac{1}{2}\right)$$

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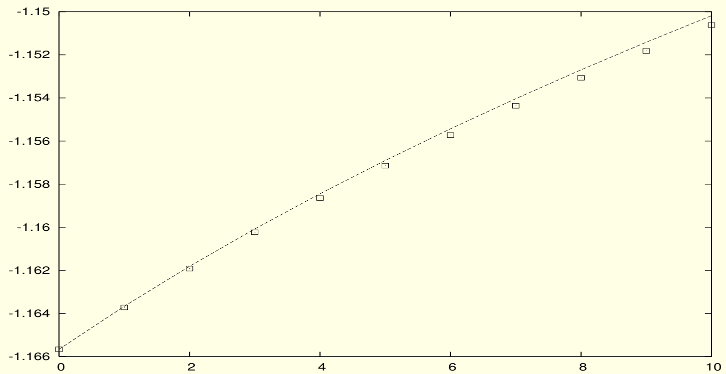

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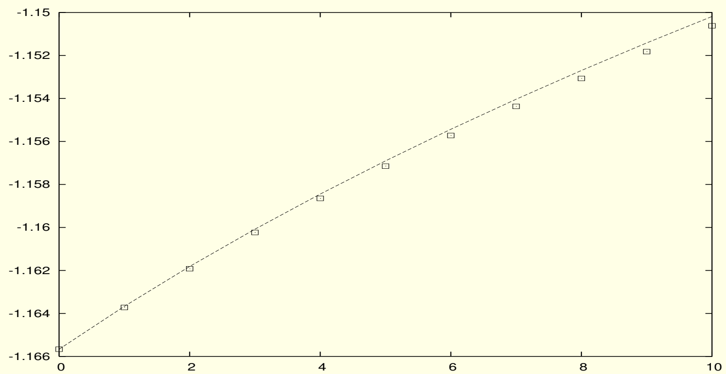

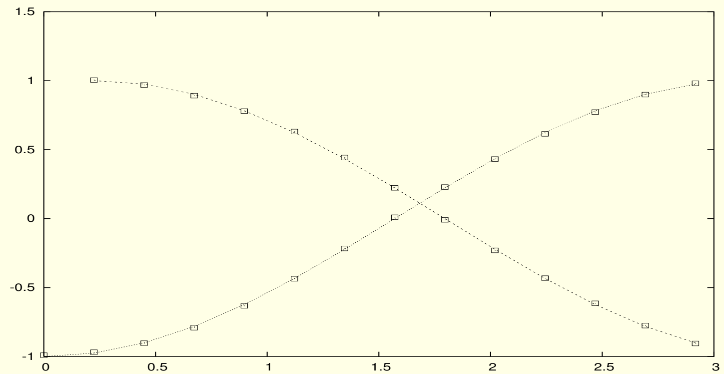

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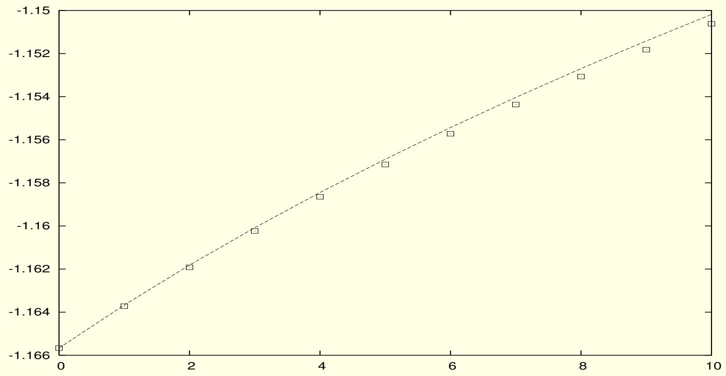

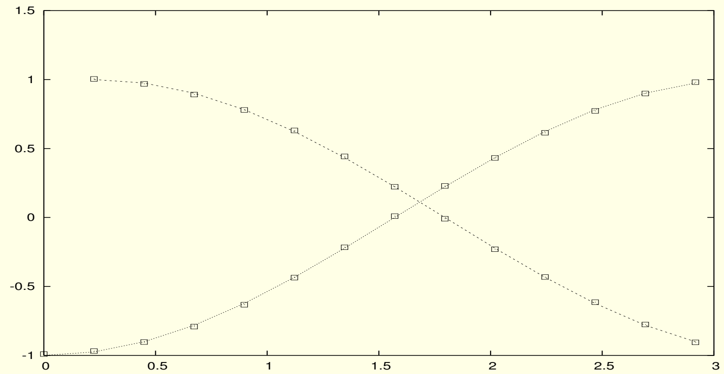

Variation as a function of $M_0=0,10$ $\qquad \frac{b\varphi_0}{2}=0,\pi$

Summary

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Periodic

Summary

Periodic
boundary

Summary

Periodic
boundary
condition

Summary

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We have calculated $E_0(L, m, \lambda)$

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Why did we need $E_0(L)$?

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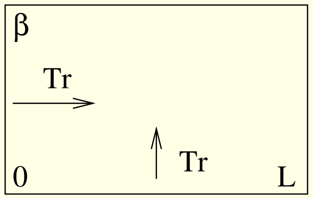

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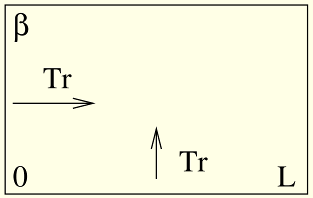

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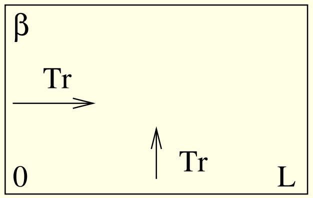

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sinh-Gordon

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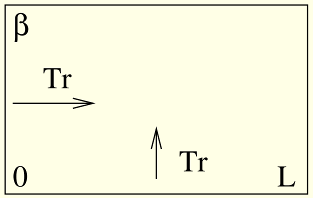

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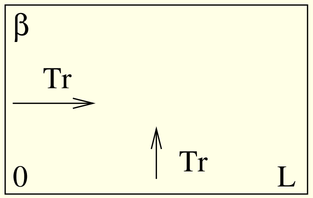

$Z = e^{-E_0(L)\beta}$ for $\beta \to \infty$

sinh-Gordon

sine-Gordon

$E_0(m,\lambda) \leftrightarrow E_0^{pert}(\mu,b)$

Summary

Periodic boundary condition

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Why did we need $E_0(L)$?
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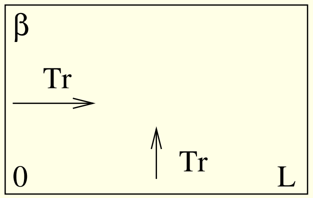

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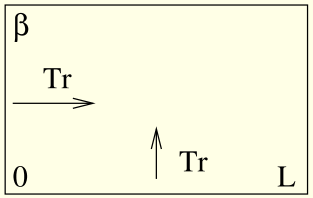

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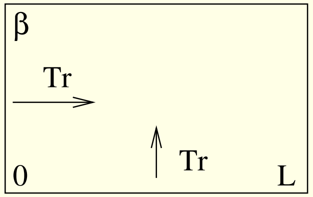

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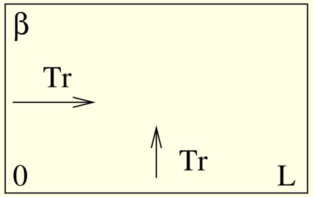

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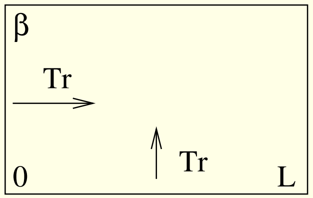

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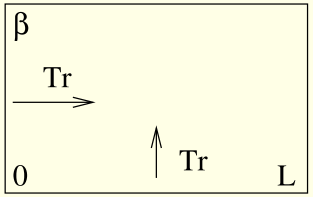

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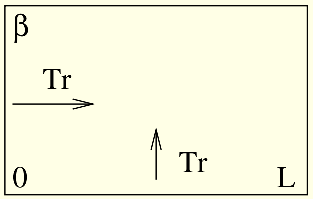

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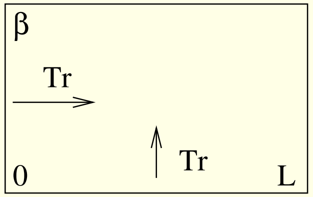

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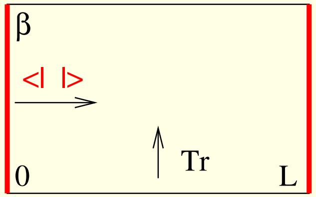

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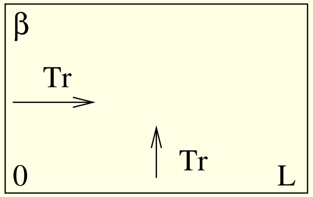

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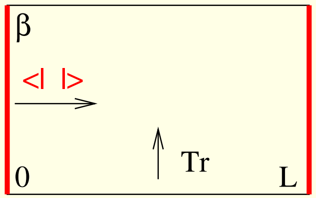

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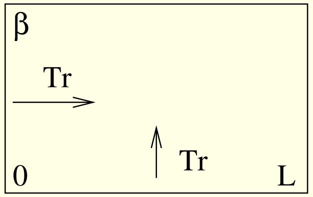

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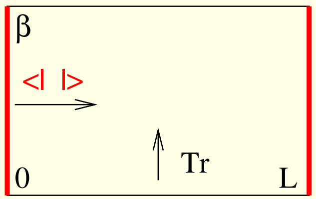

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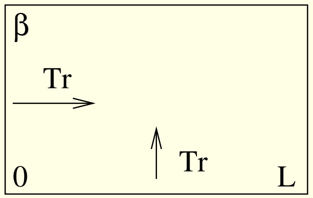

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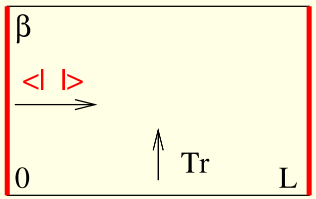

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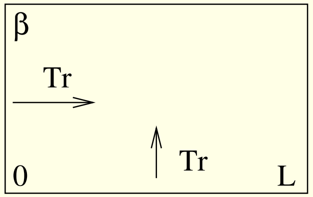

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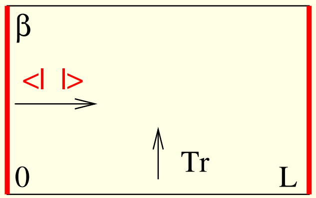

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