JSTATP06010
hep-th/0601107

Workshop and Summer School: From Statistical Mechanics to Conformal and Quantum Field Theory

8 January - 8 February, 2007, Melbourne, Australia

Equivalences between spin models induced by defects

Zoltán Bajnok

Eötvös University, Budapest

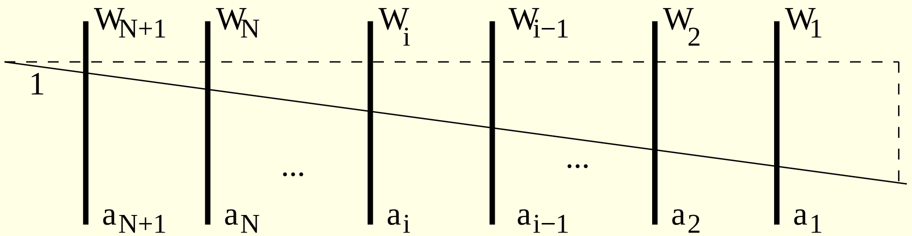
JSTATP06010
hep-th/0601107

Workshop and Summer School: From Statistical Mechanics to Conformal and Quantum Field Theory
8 January - 8 February, 2007, Melbourne, Australia

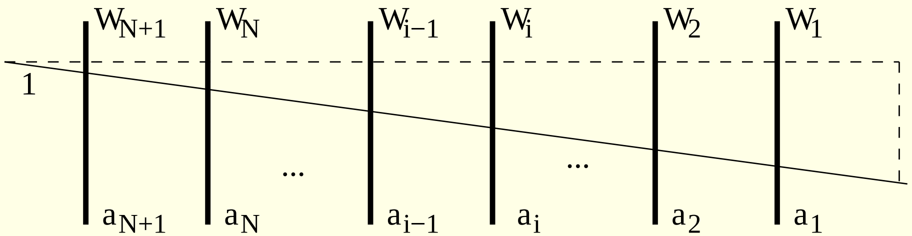
Equivalences between spin models induced by defects

Zoltán Bajnok
Eötvös University, Budapest

Closed spin chain



Equivalent closed spin chain



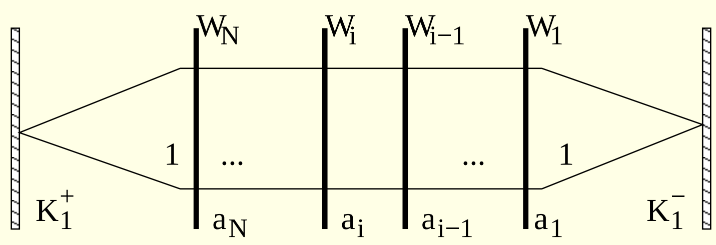
JSTATP06010
hep-th/0601107

Workshop and Summer School: From Statistical Mechanics to Conformal and Quantum Field Theory
8 January - 8 February, 2007, Melbourne, Australia

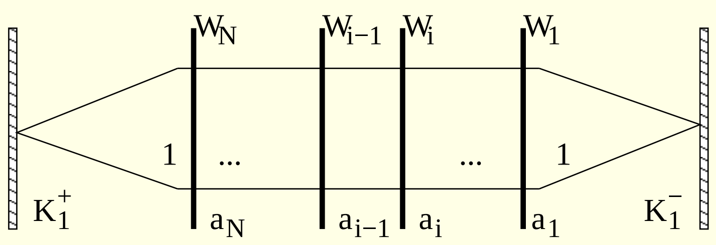
Equivalences between spin models induced by defects

Zoltán Bajnok
Eötvös University, Budapest

Open spin chain



Equivalent open spin chain

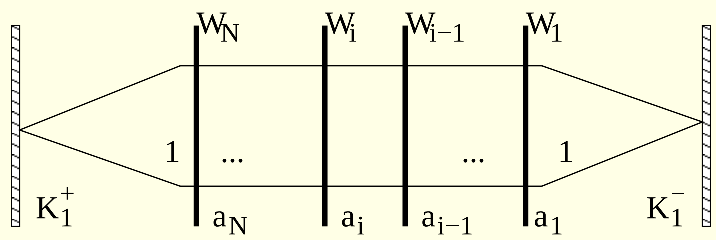


JSTATP06010
hep-th/0601107
Workshop and Summer School: From Statistical Mechanics to Conformal and Quantum Field Theory
8 January - 8 February, 2007, Melbourne, Australia

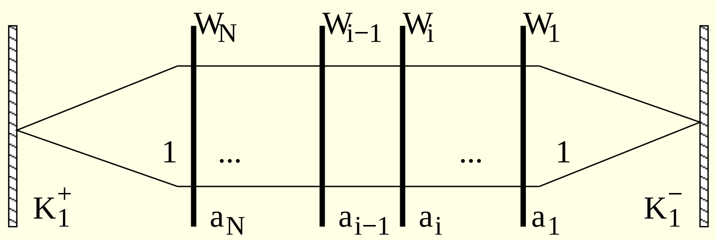
Equivalences between spin models induced by defects

Zoltán Bajnok
Eötvös University, Budapest

Open spin chain

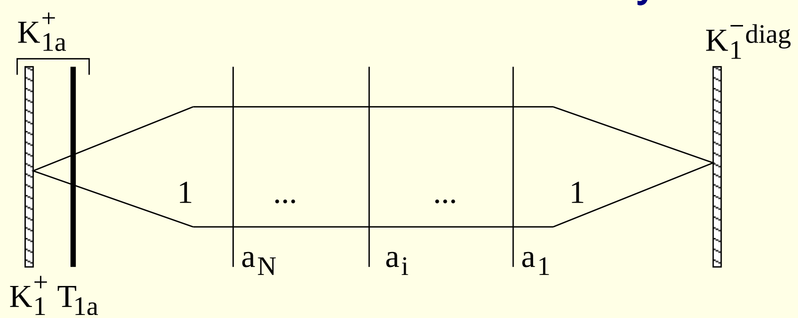


Equivalent open spin chain

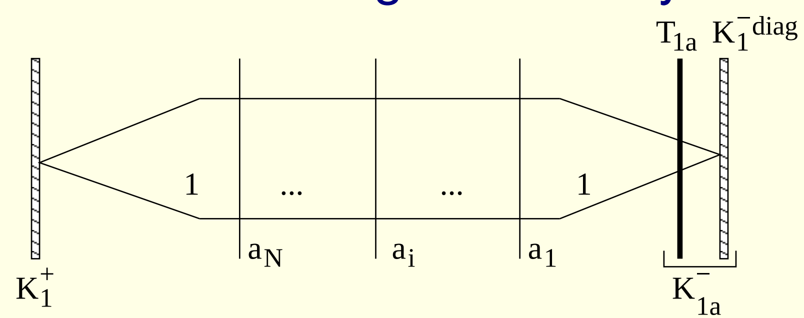


Consequence: equivalences between different boundary conditions

Dressed left boundary



Dressed right boundary



Motivation: the two boundary XXZ

Motivation: the two boundary XXZ

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N-1}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \cosh\eta\,\sigma_n^x\sigma_{n+1}^x) +$$

Motivation: the two boundary XXZ

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N-1}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \cosh\eta\,\sigma_n^x\sigma_{n+1}^x) + (\sigma_N^x\sigma_1^x + \sigma_N^y\sigma_1^y + \cosh\eta\,\sigma_N^x\sigma_1^x)$$

Motivation: the two boundary XXZ

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N-1}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \cosh\eta\,\sigma_n^x\sigma_{n+1}^x) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + (1 \leftrightarrow N)$$

Motivation: the two boundary XXZ

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N-1}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \cosh\eta\,\sigma_n^x\sigma_{n+1}^x) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + (1 \leftrightarrow N)$$

$$\gamma_{1,N}^z = \pm\frac{1}{2}\sinh\eta\coth\widehat{\alpha}_\mp\tanh\widehat{\beta}_\mp\,; \quad \gamma_{1,N}^x = \frac{\sinh\eta\cosh\widehat{\theta}_\mp}{2\sinh\widehat{\alpha}_\mp\cosh\widehat{\beta}_\mp}\,; \quad \gamma_{1,N}^y = \frac{i\sinh\eta\sinh\widehat{\theta}_\mp}{2\sinh\widehat{\alpha}_\mp\cosh\widehat{\beta}_\mp}$$

Motivation: the two boundary XXZ

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N-1}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \cosh\eta\,\sigma_n^x\sigma_{n+1}^x) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + (1 \leftrightarrow N)$$

$$\gamma_{1,N}^z = \pm\frac{1}{2}\sinh\eta\coth\widehat{\alpha}_\mp\tanh\widehat{\beta}_\mp\,; \quad \gamma_{1,N}^x = \frac{\sinh\eta\cosh\widehat{\theta}_\mp}{2\sinh\widehat{\alpha}_\mp\cosh\widehat{\beta}_\mp}\,; \quad \gamma_{1,N}^y = \frac{i\sinh\eta\sinh\widehat{\theta}_\mp}{2\sinh\widehat{\alpha}_\mp\cosh\widehat{\beta}_\mp}$$

Diagonal solution (BA=reference state) $\gamma_{1,N}^x = \gamma_{1,N}^y = 0$: $\widehat{\beta}_\pm \to \infty$

F. C. Alcaraz, M. Barber, M. T. Batchelor, R. J. Baxter, G. R. W. Quispel, *J. Phys.* **A**20, 6397 (1987)

Motivation: the two boundary XXZ

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N-1}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \cosh\eta\,\sigma_n^x\sigma_{n+1}^x) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + (1 \leftrightarrow N)$$

$$\gamma_{1,N}^z = \pm\frac{1}{2}\sinh\eta\coth\widehat{\alpha}_\mp\tanh\widehat{\beta}_\mp\,; \quad \gamma_{1,N}^x = \frac{\sinh\eta\cosh\widehat{\theta}_\mp}{2\sinh\widehat{\alpha}_\mp\cosh\widehat{\beta}_\mp}\,; \quad \gamma_{1,N}^y = \frac{i\sinh\eta\sinh\widehat{\theta}_\mp}{2\sinh\widehat{\alpha}_\mp\cosh\widehat{\beta}_\mp}$$

Diagonal solution (BA=reference state) $\gamma_{1,N}^x = \gamma_{1,N}^y = 0$: $\widehat{\beta}_\pm \to \infty$

F. C. Alcaraz, M. Barber, M. T. Batchelor, R. J. Baxter, G. R. W. Quispel, *J. Phys.* **A**20, 6397 (1987)

Nondiagonal, constrained solution (BA): $\widehat{\alpha}_- + \widehat{\beta}_- + \widehat{\alpha}_+ + \widehat{\beta}_+ = \widehat{\theta}_- - \widehat{\theta}_+ + \eta$

R.I. Nepomechie, *J. Stat. Phys.* **111** 1363 (2003), *J.Phys.* **A37** 433 (2004) (root of unity)

J. Cao, H.-Q. Lin, K.-J. Shi, Y. Wang, *Nucl. Phys.* **B663** 487 (2003)

Motivation: the two boundary XXZ

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N-1}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \cosh\eta\,\sigma_n^x\sigma_{n+1}^x) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + (1 \leftrightarrow N)$$

$$\gamma_{1,N}^z = \pm\frac{1}{2}\sinh\eta\coth\widehat{\alpha}_\mp\tanh\widehat{\beta}_\mp\,; \quad \gamma_{1,N}^x = \frac{\sinh\eta\cosh\widehat{\theta}_\mp}{2\sinh\widehat{\alpha}_\mp\cosh\widehat{\beta}_\mp}\,; \quad \gamma_{1,N}^y = \frac{i\sinh\eta\sinh\widehat{\theta}_\mp}{2\sinh\widehat{\alpha}_\mp\cosh\widehat{\beta}_\mp}$$

Diagonal solution (BA=reference state) $\gamma_{1,N}^x = \gamma_{1,N}^y = 0$: $\widehat{\beta}_\pm \to \infty$

F. C. Alcaraz, M. Barber, M. T. Batchelor, R. J. Baxter, G. R. W. Quispel, *J. Phys.* **A**20, 6397 (1987)

Nondiagonal, constrained solution (BA): $\widehat{\alpha}_- + \widehat{\beta}_- + \widehat{\alpha}_+ + \widehat{\beta}_+ = \widehat{\theta}_- - \widehat{\theta}_+ + \eta$

R.I. Nepomechie, *J. Stat. Phys.* **111** 1363 (2003), *J.Phys.* **A37** 433 (2004) (root of unity)

J. Cao, H.-Q. Lin, K.-J. Shi, Y. Wang, *Nucl. Phys.* **B663** 487 (2003)

Constrained solution (BA)+spectral equivalences between different BC-s:

J. de Gier, P. Pyatov, *JSTAT* **0403** P002 (2004)

A. Nichols, V. Rittenberg, J. de Gier, *J. Stat. Mech.* P03003 (2005)

Application: the two boundary XXZ

Application: the two boundary XXZ

Non-equilibrum statistical model of the asymmetric exclusion process:

J. de Gier, F. H. L. Essler, *Phys. Rev. Lett.* **95**, 240601 (2005)

Application: the two boundary XXZ

Non-equilibrum statistical model of the asymmetric exclusion process:

J. de Gier, F. H. L. Essler, *Phys. Rev. Lett.* **95**, 240601 (2005)

Raise and Peel models

P Pyatov, *JSTAT* P09003 (2004)

Application: the two boundary XXZ

Non-equilibrum statistical model of the asymmetric exclusion process:

J. de Gier, F. H. L. Essler, *Phys. Rev. Lett.* **95**, 240601 (2005)

Raise and Peel models

P Pyatov, *JSTAT* P09003 (2004)

Lattice sine-Gordon, lattice Liouville

A. Doikou, *J. Stat. Mech. (2006) P09010*

Application: the two boundary XXZ

Non-equilibrum statistical model of the asymmetric exclusion process:

J. de Gier, F. H. L. Essler, *Phys. Rev. Lett.* **95**, 240601 (2005)

Raise and Peel models

P Pyatov, *JSTAT* P09003 (2004)

Lattice sine-Gordon, lattice Liouville

A. Doikou, *J. Stat. Mech. (2006) P09010*

Sine-Gordon field theory on the strip

Sergei Skorik, Hubert Saleur, *J.Phys.***A28**:6605-6622,1995

Changrim Ahn , M. Bellacosa, F. Ravanini, *Phys.Lett.***B595**:537-546,2004

Changrim Ahn, Rafael I. Nepomechie, *Nucl.Phys.***B676**:637-658,2004

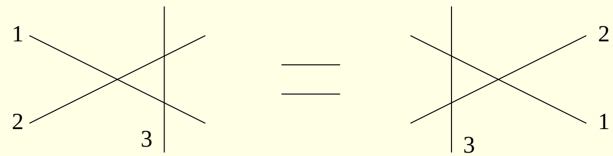
Changrim Ahn, Z. Bajnok, R. I. Nepomechie, L. Palla, G. Takacs,*Nucl.Phys.***B714**:307-335,2005

Building up the closed model

Building up the closed model

Y-B eq. (symmetric, unitary, crossing symmetric) solution on $V \otimes V$

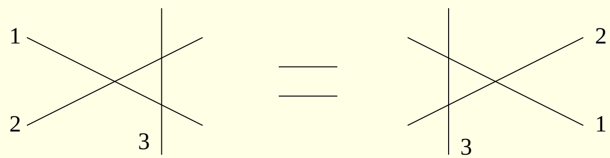
$$R_{12}(u_1 - u_2)R_{13}(u_1 - u_3)R_{23}(u_2 - u_3) \quad = \quad R_{23}(u_2 - u_3)R_{13}(u_1 - u_3)R_{12}(u_1 - u_2)$$



Building up the closed model

Y-B eq. (symmetric, unitary, crossing symmetric) solution on $V \otimes V$

$$R_{12}(u_1 - u_2) R_{13}(u_1 - u_3) R_{23}(u_2 - u_3) \quad = \quad R_{23}(u_2 - u_3) R_{13}(u_1 - u_3) R_{12}(u_1 - u_2)$$

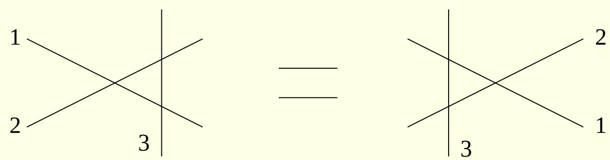


$$R_{12}(u_1 - u_2) = \pi_1^{u_1} \otimes \pi_2^{u_2}(\mathcal{R})$$

Building up the closed model

Y-B eq. (symmetric, unitary, crossing symmetric) solution on $V \otimes V$

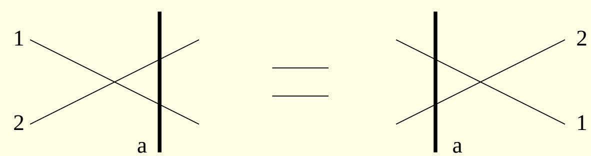
$$R_{12}(u_1 - u_2)R_{13}(u_1 - u_3)R_{23}(u_2 - u_3) \;=\; R_{23}(u_2 - u_3)R_{13}(u_1 - u_3)R_{12}(u_1 - u_2)$$



$$R_{12}(u_1 - u_2) = \pi_1^{u_1} \otimes \pi_2^{u_2}(\mathcal{R})$$

RTT eq. with a solution in the quantum space W_a

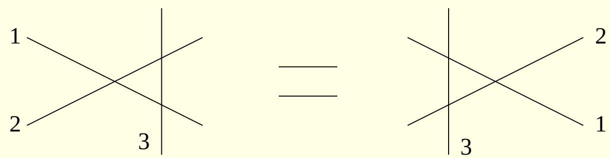
$$R_{12}(u_1 - u_2)T_{1a}(u_1)T_{2a}(u_2) = T_{2a}(u_2)T_{1a}(u_1)R_{12}(u_1 - u_2)$$



Building up the closed model

Y-B eq. (symmetric, unitary, crossing symmetric) solution on $V \otimes V$

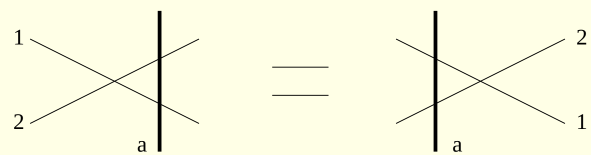
$$R_{12}(u_1 - u_2)R_{13}(u_1 - u_3)R_{23}(u_2 - u_3) \quad = \quad R_{23}(u_2 - u_3)R_{13}(u_1 - u_3)R_{12}(u_1 - u_2)$$



$$R_{12}(u_1 - u_2) = \pi_1^{u_1} \otimes \pi_2^{u_2}(\mathcal{R})$$

RTT eq. with a solution in the quantum space $W_a (= V, T_{13}(u_1) = R_{13}(u_1 - u_3))$

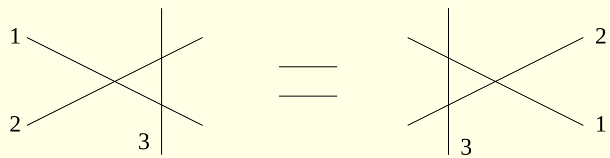
$$R_{12}(u_1 - u_2)T_{1a}(u_1)T_{2a}(u_2) = T_{2a}(u_2)T_{1a}(u_1)R_{12}(u_1 - u_2)$$



Building up the closed model

Y-B eq. (symmetric, unitary, crossing symmetric) solution on $V \otimes V$

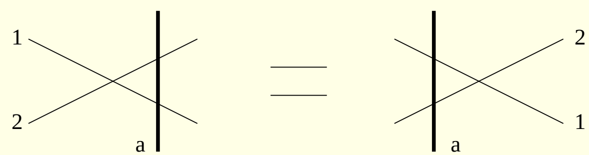
$$R_{12}(u_1 - u_2) R_{13}(u_1 - u_3) R_{23}(u_2 - u_3) \quad = \quad R_{23}(u_2 - u_3) R_{13}(u_1 - u_3) R_{12}(u_1 - u_2)$$



$$R_{12}(u_1 - u_2) = \pi_1^{u_1} \otimes \pi_2^{u_2}(\mathcal{R})$$

RTT eq. with a solution in the quantum space $W_a (= V, T_{13}(u_1) = R_{13}(u_1 - u_3))$

$$R_{12}(u_1 - u_2) T_{1a}(u_1) T_{2a}(u_2) = T_{2a}(u_2) T_{1a}(u_1) R_{12}(u_1 - u_2)$$

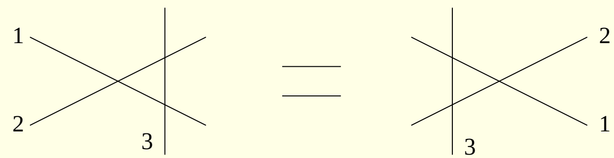


$$T_{1a}(u_1) = \pi_1^{u_1} \otimes \pi_a(\mathcal{R})$$

Building up the closed model

Y-B eq. (symmetric, unitary, crossing symmetric) solution on $V \otimes V$

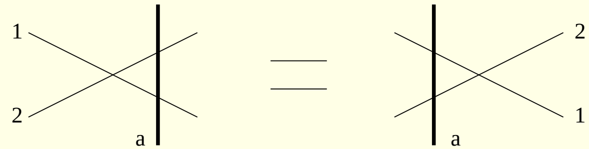
$$R_{12}(u_1 - u_2)R_{13}(u_1 - u_3)R_{23}(u_2 - u_3) = R_{23}(u_2 - u_3)R_{13}(u_1 - u_3)R_{12}(u_1 - u_2)$$



$$R_{12}(u_1 - u_2) = \pi_1^{u_1} \otimes \pi_2^{u_2}(\mathcal{R})$$

RTT eq. with a solution in the quantum space $W_a (= V, T_{13}(u_1) = R_{13}(u_1 - u_3))$

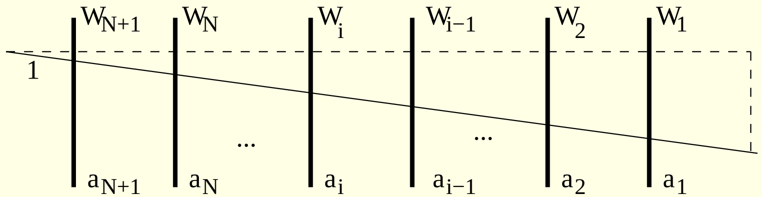
$$R_{12}(u_1 - u_2)T_{1a}(u_1)T_{2a}(u_2) = T_{2a}(u_2)T_{1a}(u_1)R_{12}(u_1 - u_2)$$



$$T_{1a}(u_1) = \pi_1^{u_1} \otimes \pi_a(\mathcal{R})$$

Transfer matrix: generating functional for conserved charges

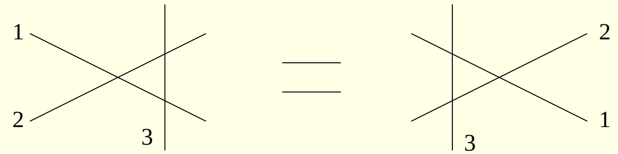
$$t(u) = \mathrm{Tr}_1(id_V T_{1-}(u)) \quad T_{1-}(u) = L_{1a_{N+1}}(u) \dots L_{1a_i}(u) \dots L_{1a_1}(u)$$



Building up the closed model

Y-B eq. (symmetric, unitary, crossing symmetric) solution on $V \otimes V$

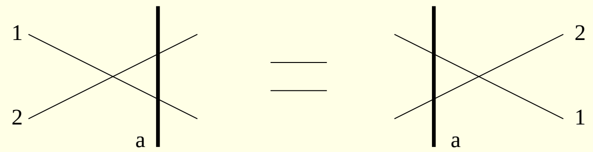
$$R_{12}(u_1-u_2)R_{13}(u_1-u_3)R_{23}(u_2-u_3) \quad = \quad R_{23}(u_2-u_3)R_{13}(u_1-u_3)R_{12}(u_1-u_2)$$



$$R_{12}(u_1-u_2)=\pi_1^{u_1}\otimes\pi_2^{u_2}(\mathcal{R})$$

RTT eq. with a solution in the quantum space $W_a (= V, T_{13}(u_1) = R_{13}(u_1-u_3))$

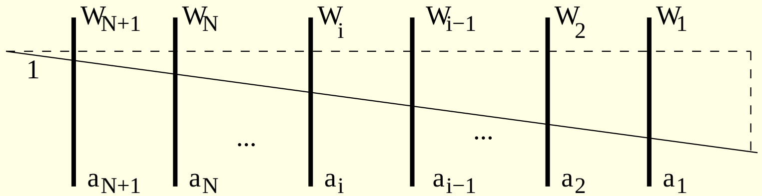
$$R_{12}(u_1-u_2)T_{1a}(u_1)T_{2a}(u_2)=T_{2a}(u_2)T_{1a}(u_1)R_{12}(u_1-u_2)$$



$$T_{1a}(u_1)=\pi_1^{u_1}\otimes\pi_a(\mathcal{R})$$

Transfer matrix: generating functional for conserved charges

$$t(u)=\mathrm{Tr}_1(id_V T_{1-}(u)) \quad T_{1-}(u)=L_{1a_{N+1}}(u)\dots L_{1a_i}(u)\dots L_{1a_1}(u)$$



Hamiltonian: $\mathcal{H} \propto (\log t)'(0)$

Building up the closed XXZ model

Building up the closed XXZ model

Yang-Baxter solution with $dimV = 2$ (quantum group: $U_q(s\hat{l}_2)$)

$$R_{12}(u) = \begin{pmatrix} \sinh(u+\eta) & 0 & 0 & 0 \\ 0 & \sinh u & \sinh \eta & 0 \\ 0 & \sinh \eta & \sinh u & 0 \\ 0 & 0 & 0 & \sinh(u+\eta) \end{pmatrix}$$

Building up the closed XXZ model

Yang-Baxter solution with $dim V = 2$ (quantum group:$U_q(\widehat{sl}_2)$)

$$R_{12}(u) = \begin{pmatrix} \sinh(u+\eta) & 0 & 0 & 0 \\ 0 & \sinh u & \sinh \eta & 0 \\ 0 & \sinh \eta & \sinh u & 0 \\ 0 & 0 & 0 & \sinh(u+\eta) \end{pmatrix}$$

RTTE solution acting in the quantum space V

$$L_{1a_i}(u) = \begin{pmatrix} \sinh(u + \frac{\eta}{2}(1+\sigma_i^z)) & \sinh \eta \, \sigma_i^- \\ \sinh \eta \, \sigma_i^+ & \sinh(u + \frac{\eta}{2}(1-\sigma_i^z)) \end{pmatrix}$$

Building up the closed XXZ model

Yang-Baxter solution with $dimV = 2$ (quantum group:$U_q(\hat{sl}_2)$)

$$R_{12}(u) = \begin{pmatrix} \sinh(u+\eta) & 0 & 0 & 0 \\ 0 & \sinh u & \sinh \eta & 0 \\ 0 & \sinh \eta & \sinh u & 0 \\ 0 & 0 & 0 & \sinh(u+\eta) \end{pmatrix}$$

RTTE solution acting in the quantum space V

$$L_{1a_i}(u) = \begin{pmatrix} \sinh(u + \frac{\eta}{2}(1+\sigma_i^z)) & \sinh \eta \, \sigma_i^- \\ \sinh \eta \, \sigma_i^+ & \sinh(u + \frac{\eta}{2}(1-\sigma_i^z)) \end{pmatrix}$$

RTT solution acting in the quantum space $dimW_a = \infty$ (q oscillator reps.)

$$T_{1a}(u,\beta) = \Gamma_1 \begin{pmatrix} e^{u+\beta} q^{-J_0} & J_- q^{J_0} \\ -J_+ q^{-J_0} & e^{u+\beta} q^{J_0} \end{pmatrix} \Gamma_2 \quad J_0 = \sum_{j=-\infty}^{\infty} j e_{jj} \quad ; \quad J_\pm = \sum_{j=-\infty}^{\infty} e_{jj\mp 1}$$

where $q = e^{-\eta}$ and $\Gamma_i = diag(e^{\gamma_i}, e^{-\gamma_i})$

Building up the closed XXZ model

Yang-Baxter solution with $dimV = 2$ (quantum group:$U_q(s\hat{l}_2)$)

$$R_{12}(u) = \begin{pmatrix} \sinh(u+\eta) & 0 & 0 & 0 \\ 0 & \sinh u & \sinh \eta & 0 \\ 0 & \sinh \eta & \sinh u & 0 \\ 0 & 0 & 0 & \sinh(u+\eta) \end{pmatrix}$$

RTTE solution acting in the quantum space V

$$L_{1a_i}(u) = \begin{pmatrix} \sinh(u + \frac{\eta}{2}(1+\sigma_i^z)) & \sinh \eta \, \sigma_i^- \\ \sinh \eta \, \sigma_i^+ & \sinh(u + \frac{\eta}{2}(1-\sigma_i^z)) \end{pmatrix}$$

RTT solution acting in the quantum space $dimW_a = \infty$ (q oscillator reps.)

$$T_{1a}(u,\beta) = \Gamma_1 \begin{pmatrix} e^{u+\beta} q^{-J_0} & J_- q^{J_0} \\ -J_+ q^{-J_0} & e^{u+\beta} q^{J_0} \end{pmatrix} \Gamma_2 \quad J_0 = \sum_{j=-\infty}^{\infty} j e_{jj} \quad ; \quad J_\pm = \sum_{j=-\infty}^{\infty} e_{jj\mp 1}$$

where $q = e^{-\eta}$ and $\Gamma_i = diag(e^{\gamma_i}, e^{-\gamma_i})$

Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(id_V T_{1-}(u)) \quad T_{1-}(u) = L_{1a_{N+1}}(u) \dots L_{1a_i}(u) \dots L_{1a_1}(u)$$

Building up the closed XXZ model

Yang-Baxter solution with $dimV = 2$ (quantum group:$U_q(\hat{sl}_2)$)

$$R_{12}(u) = \begin{pmatrix} \sinh(u+\eta) & 0 & 0 & 0 \\ 0 & \sinh u & \sinh\eta & 0 \\ 0 & \sinh\eta & \sinh u & 0 \\ 0 & 0 & 0 & \sinh(u+\eta) \end{pmatrix}$$

RTTE solution acting in the quantum space V

$$L_{1a_i}(u) = \begin{pmatrix} \sinh(u + \frac{\eta}{2}(1+\sigma_i^z)) & \sinh\eta\,\sigma_i^- \\ \sinh\eta\,\sigma_i^+ & \sinh(u+\frac{\eta}{2}(1-\sigma_i^z)) \end{pmatrix}$$

RTT solution acting in the quantum space $dimW_a = \infty$ (q oscillator reps.)

$$T_{1a}(u,\beta) = \Gamma_1 \begin{pmatrix} e^{u+\beta}q^{-J_0} & J_- q^{J_0} \\ -J_+ q^{-J_0} & e^{u+\beta} q^{J_0} \end{pmatrix} \Gamma_2 \quad J_0 = \sum_{j=-\infty}^{\infty} j e_{jj} \quad ; \quad J_\pm = \sum_{j=-\infty}^{\infty} e_{jj\mp 1}$$

where $q = e^{-\eta}$ and $\Gamma_i = diag(e^{\gamma_i}, e^{-\gamma_i})$

Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(id_V T_{1-}(u)) \quad T_{1-}(u) = L_{1a_{N+1}}(u) \dots L_{1a_i}(u) \dots L_{1a_1}(u)$$

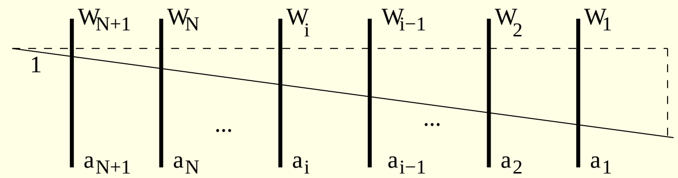
Hamiltonian: $\mathcal{H} = (\log t)'(0) \propto \frac{1}{2}\sum_{n=1}^{N}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \cosh\eta\,\sigma_n^x\sigma_{n+1}^x)$

Equivalence in closed chains: spin order is spectrally irrelevant

Equivalence in closed chains: spin order is spectrally irrelevant

Transfer matrix: generating functional for conserved charges

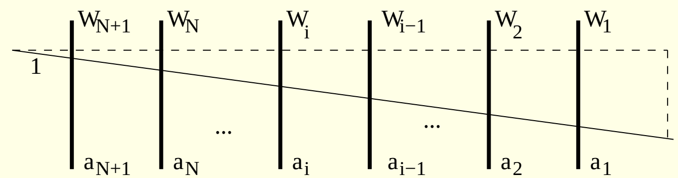
$$t(u) = \mathrm{Tr}_1(L^{N+1}_{1a_{N+1}}(u)\ldots L^{i+1}_{1a_{i+1}}(u)L^{i}_{1a_i}(u)L^{i-1}_{1a_{i-1}}(u)L^{i-2}_{1a_{i-2}}(u)\ldots L^{1}_{1a_1}(u))$$



Equivalence in closed chains: spin order is spectrally irrelevant

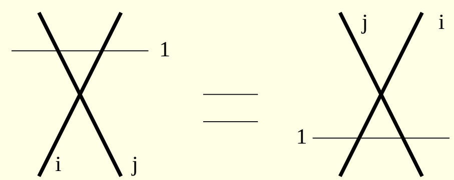
Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(L^{N+1}_{1a_{N+1}}(u)\ldots L^{i+1}_{1a_{i+1}}(u)L^{i}_{1a_i}(u)L^{i-1}_{1a_{i-1}}(u)L^{i-2}_{1a_{i-2}}(u)\ldots L^{1}_{1a_1}(u))$$



SLL equation: suppose there exists $S_{a_ia_j}(u_i-u_j) \in End(W_i \otimes W_j)$ such that

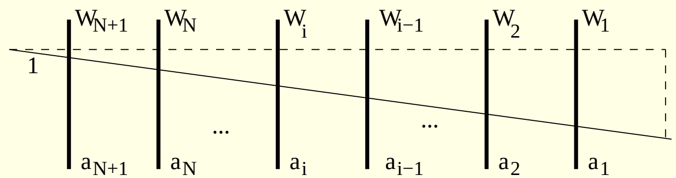
$$S_{a_ia_j}(u_i-u_j)L^{i}_{1a_i}(u_i)L^{j}_{1a_j}(u_j) \;=\; L^{j}_{1a_j}(u_j)L^{i}_{1a_i}(u_i)S_{a_ia_j}(u_i-u_j)$$



Equivalence in closed chains: spin order is spectrally irrelevant

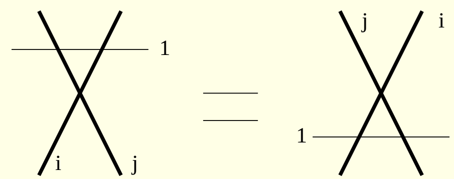
Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(L^{N+1}_{1a_{N+1}}(u)\dots L^{i+1}_{1a_{i+1}}(u)L^{i}_{1a_{i}}(u)L^{i-1}_{1a_{i-1}}(u)L^{i-2}_{1a_{i-2}}(u)\dots L^{1}_{1a_{1}}(u))$$



SLL equation: suppose there exists $S_{a_ia_j}(u_i - u_j) \in End(W_i \otimes W_j)$ such that

$$S_{a_ia_j}(u_i-u_j)L^{i}_{1a_i}(u_i)L^{j}_{1a_j}(u_j) \quad = \quad L^{j}_{1a_j}(u_j)L^{i}_{1a_i}(u_i)S_{a_ia_j}(u_i-u_j)$$

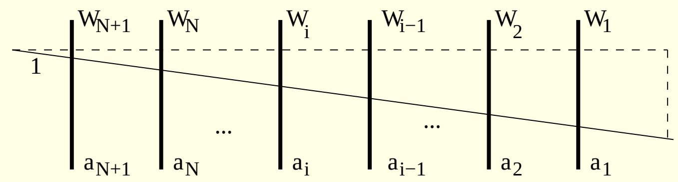


$$S_{ab} = \pi_a \otimes \pi_b(\mathcal{R})$$

Equivalence in closed chains: spin order is spectrally irrelevant

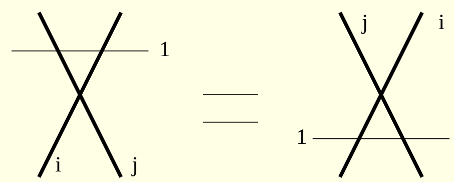
Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(L^{N+1}_{1a_{N+1}}(u)\dots L^{i+1}_{1a_{i+1}}(u)L^{i}_{1a_{i}}(u)L^{i-1}_{1a_{i-1}}(u)L^{i-2}_{1a_{i-2}}(u)\dots L^{1}_{1a_{1}}(u))$$



SLL equation: suppose there exists $S_{a_ia_j}(u_i-u_j) \in End(W_i \otimes W_j)$ such that

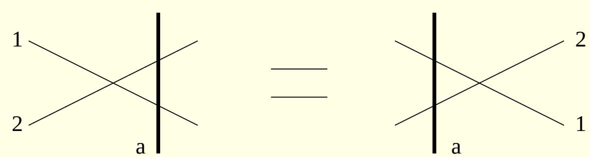
$$S_{a_ia_j}(u_i-u_j)L^{i}_{1a_i}(u_i)L^{j}_{1a_j}(u_j) \;=\; L^{j}_{1a_j}(u_j)L^{i}_{1a_i}(u_i)S_{a_ia_j}(u_i-u_j)$$



$$S_{ab} = \pi_a \otimes \pi_b(\mathcal{R})$$

Example: RTT equation $R_{12} \to L_{12}$, $T_{1a} \to L_{1a}$, and $T^{-1}_{2a} \to S_{2a}$

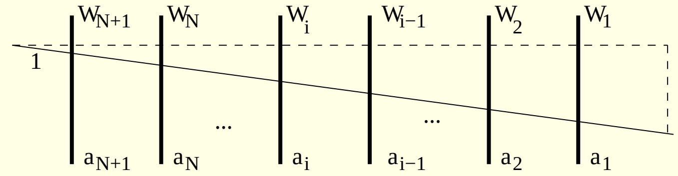
$$T_{2a}(u_2)^{-1}R_{12}(u_1-u_2)T_{1a}(u_1) = T_{1a}(u_1)R_{12}(u_1-u_2)T_{2a}(u_2)^{-1}$$



Equivalence in closed chains: spin order is spectrally irrelevant

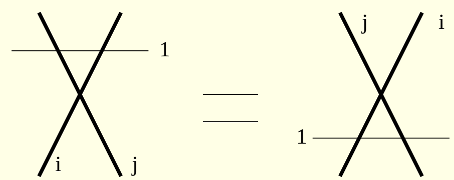
Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(L^{N+1}_{1a_{N+1}}(u)\ldots L^{i+1}_{1a_{i+1}}(u)L^{i}_{1a_{i}}(u)L^{i-1}_{1a_{i-1}}(u)L^{i-2}_{1a_{i-2}}(u)\ldots L^{1}_{1a_{1}}(u))$$



SLL equation: suppose there exists $S_{a_ia_j}(u_i-u_j)\in End(W_i\otimes W_j)$ such that

$$S_{a_ia_j}(u_i-u_j)L^{i}_{1a_i}(u_i)L^{j}_{1a_j}(u_j) \;=\; L^{j}_{1a_j}(u_j)L^{i}_{1a_i}(u_i)S_{a_ia_j}(u_i-u_j)$$



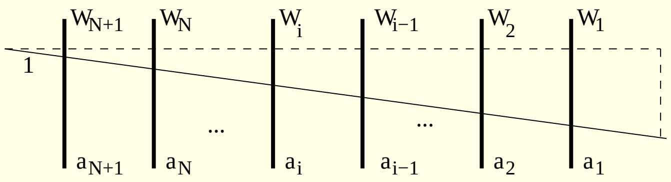
$$S_{ab} = \pi_a\otimes\pi_b(\mathcal{R})$$

Exchange $L^{i}_{1a_i}(u)L^{i-1}_{1a_{i-1}}(u)$ with $S^{-1}_{a_ia_{i-1}}(0)L^{i-1}_{1a_{i-1}}(u)L^{i}_{1a_i}(u)S_{a_ia_{i-1}}(0)$:

Equivalence in closed chains: spin order is spectrally irrelevant

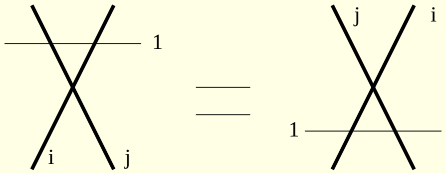
Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(L^{N+1}_{1a_{N+1}}(u) \ldots L^{i+1}_{1a_{i+1}}(u) L^{i}_{1a_i}(u) L^{i-1}_{1a_{i-1}}(u) L^{i-2}_{1a_{i-2}}(u) \ldots L^{1}_{1a_1}(u))$$



SLL equation: suppose there exists $S_{a_i a_j}(u_i - u_j) \in End(W_i \otimes W_j)$ such that

$$S_{a_i a_j}(u_i - u_j) L^i_{1a_i}(u_i) L^j_{1a_j}(u_j) \;=\; L^j_{1a_j}(u_j) L^i_{1a_i}(u_i) S_{a_i a_j}(u_i - u_j)$$

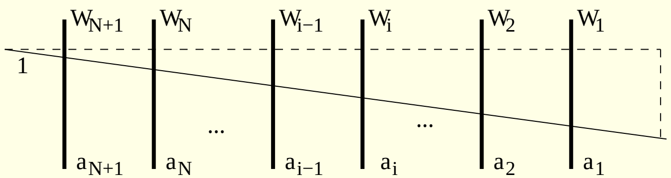


$$S_{ab} = \pi_a \otimes \pi_b(\mathcal{R})$$

Exchange $L^i_{1a_i}(u) L^{i-1}_{1a_{i-1}}(u)$ with $S^{-1}_{a_i a_{i-1}}(0) L^{i-1}_{1a_{i-1}}(u) L^i_{1a_i}(u) S_{a_i a_{i-1}}(0)$:

Equivalent chain:

$$S\, t(u)\, S^{-1} = \mathrm{Tr}_1(L^{N+1}_{1a_{N+1}}(u) \ldots L^{i+1}_{1a_{i+1}}(u) L^{i-1}_{1a_{i-1}}(u) L^{i}_{1a_i}(u) L^{i-2}_{1a_{i-2}}(u) \ldots L^{1}_{1a_1}(u))$$

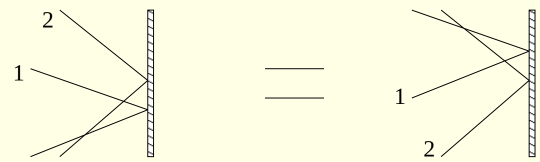


Building up the open spin chain

Building up the open spin chain

Boundary Yang-Baxter equation

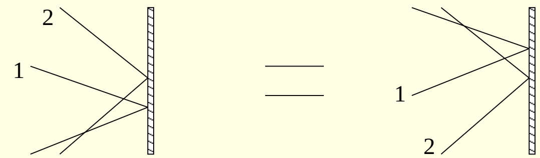
$$R_{12}(u_1 - u_2)\mathcal{T}_1^-(u_1)R_{12}(u_1 + u_2)\mathcal{T}_2^-(u_2) = \mathcal{T}_2^-(u_2)R_{12}(u_1 + u_2)\mathcal{T}_1^-(u_1)R_{12}(u_1 - u_2)$$



Building up the open spin chain

Boundary Yang-Baxter equation

$$R_{12}(u_1-u_2)\mathcal{T}_1^-(u_1)R_{12}(u_1+u_2)\mathcal{T}_2^-(u_2)=\mathcal{T}_2^-(u_2)R_{12}(u_1+u_2)\mathcal{T}_1^-(u_1)R_{12}(u_1-u_2)$$

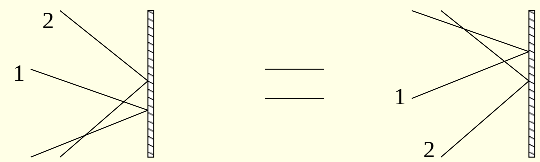


Boundary Yang-Baxter solution (left boundary) $\mathcal{T}^+(u)=\mathcal{T}^-(-u-\eta)^t$

Building up the open spin chain

Boundary Yang-Baxter equation

$$R_{12}(u_1-u_2)\mathcal{T}_1^-(u_1)R_{12}(u_1+u_2)\mathcal{T}_2^-(u_2) = \mathcal{T}_2^-(u_2)R_{12}(u_1+u_2)\mathcal{T}_1^-(u_1)R_{12}(u_1-u_2)$$

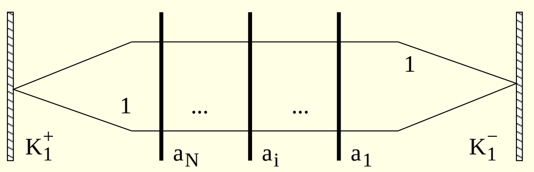


Boundary Yang-Baxter solution (left boundary) $\mathcal{T}^+(u) = \mathcal{T}^-(-u-\eta)^t$

Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(\mathcal{T}_1^+(u)\mathcal{T}_1^-(u)) \quad ; \qquad \mathcal{T}_1^+(u) = K_1^+(u)\,, \quad \mathcal{T}_1^-(u) = T_{1-}(u)K_1^-(u)T_{1-}^{-1}(-u))$$

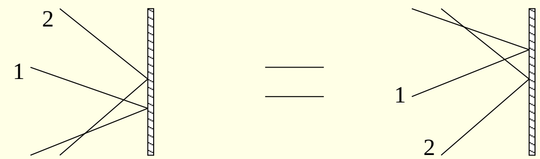
$T_{1-}(u) = L_{1a_{N+1}}(u)\ldots L_{1a_i}(u)\ldots L_{1a_1}(u)$ solution of the RTT equation



Building up the open spin chain

Boundary Yang-Baxter equation

$$R_{12}(u_1-u_2)\mathcal{T}_1^-(u_1)R_{12}(u_1+u_2)\mathcal{T}_2^-(u_2)=\mathcal{T}_2^-(u_2)R_{12}(u_1+u_2)\mathcal{T}_1^-(u_1)R_{12}(u_1-u_2)$$

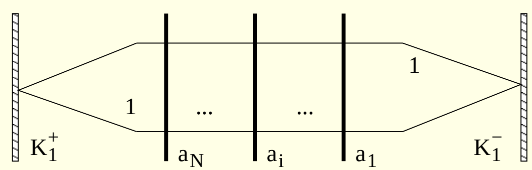


Boundary Yang-Baxter solution (left boundary) $\mathcal{T}^+(u)=\mathcal{T}^-(-u-\eta)^t$

Transfer matrix: generating functional for conserved charges

$$t(u)=\mathrm{Tr}_1(\mathcal{T}_1^+(u)\mathcal{T}_1^-(u)) \quad ; \qquad \mathcal{T}_1^+(u)=K_1^+(u)\,, \quad \mathcal{T}_1^-(u)=T_{1-}(u)K_1^-(u)T_{1-}^{-1}(-u))$$

$T_{1-}(u)=L_{1a_{N+1}}(u)\dots L_{1a_i}(u)\dots L_{1a_1}(u)$ solution of the RTT equation



Hamiltonian: $\mathcal{H}\propto t'(0)$

Building up the boundary XXZ model

Building up the boundary XXZ model

Trivial BYBE solution $K_1^-(u)^{QGI} = diag(e^u, e^{-u})$

Building up the boundary XXZ model

Trivial BYBE solution $K_1^-(u)^{QGI} = diag(e^u, e^{-u})$

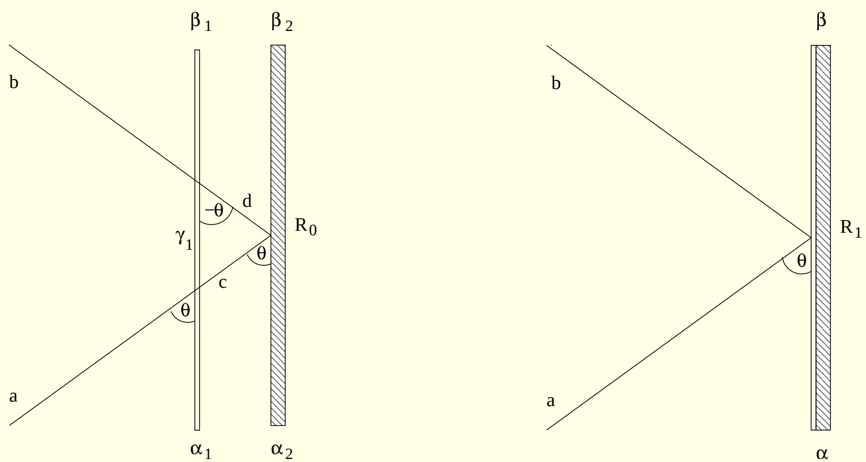
General diagonal solution by dressing

$$K_1^-(u,\alpha)^{diag} = T_{1a}(u,\alpha)K_1^{-QGI}T_{1a}^{-1}(-u,\alpha) \propto \begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix} = \begin{pmatrix} \cosh(u+\alpha) & 0 \\ 0 & \cosh(u-\alpha) \end{pmatrix}$$

Building up the boundary XXZ model

Trivial BYBE solution $K_1^-(u)^{QGI} = diag(e^u, e^{-u})$

General diagonal solution by dressing



$$T_{1a}(u,\alpha) = \Gamma_1 \begin{pmatrix} e^{u+\alpha} q^{-J_0} & J_- q^{J_0} \\ -J_+ q^{-J_0} & e^{u+\alpha} q^{J_0} \end{pmatrix} \Gamma_2$$

$$K_1^-(u,\alpha)^{diag} = T_{1a}(u,\alpha) K_1^{-QGI} T_{1a}^{-1}(-u,\alpha) \propto \begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix} = \begin{pmatrix} \cosh(u+\alpha) & 0 \\ 0 & \cosh(u-\alpha) \end{pmatrix}$$

Building up the boundary XXZ model

Trivial BYBE solution $K_1^-(u)^{QGI} = diag(e^u, e^{-u})$

General diagonal solution by dressing

$$K_1^-(u,\alpha)^{diag} = T_{1a}(u,\alpha) K_1^{-QGI} T_{1a}^{-1}(-u,\alpha) \propto \begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix} = \begin{pmatrix} \cosh(u+\alpha) & 0 \\ 0 & \cosh(u-\alpha) \end{pmatrix}$$

General nondiagonal solution acting in the quantum space $dimW_a = \infty$

$$K_{1a}^-(u,\alpha,\beta,\gamma) = T_{1a}(u,\beta) K_1^-(u,\alpha)^{diag} T_{1a}^{-1}(-u,\beta) \propto \begin{pmatrix} e^{\beta}P_+ + e^{-\beta}P_- & -J_- e^{i\gamma}\sinh 2u \\ J_+ e^{-i\gamma}\sinh 2u & e^{\beta}P_- + e^{-\beta}P_+ \end{pmatrix}$$

scalar nondiagonal solution $K_1^-(u,\alpha,\beta,\gamma) = K_{1a}^-(u,\alpha,\beta,\gamma, J_\pm \to e^{\mp i\theta})$

Building up the boundary XXZ model

Trivial BYBE solution $K_1^-(u)^{QGI} = diag(e^u, e^{-u})$

General diagonal solution by dressing

$$K_1^-(u,\alpha)^{diag} = T_{1a}(u,\alpha) K_1^{-QGI} T_{1a}^{-1}(-u,\alpha) \propto \begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix} = \begin{pmatrix} \cosh(u+\alpha) & 0 \\ 0 & \cosh(u-\alpha) \end{pmatrix}$$

General nondiagonal solution acting in the quantum space $dimW_a = \infty$

$$K_{1a}^-(u,\alpha,\beta,\gamma) = T_{1a}(u,\beta) K_1^-(u,\alpha)^{diag} T_{1a}^{-1}(-u,\beta) \propto \begin{pmatrix} e^\beta P_+ + e^{-\beta} P_- & -J_- e^{i\gamma} \sinh 2u \\ J_+ e^{-i\gamma} \sinh 2u & e^\beta P_- + e^{-\beta} P_+ \end{pmatrix}$$

scalar nondiagonal solution $K_1^-(u,\alpha,\beta,\gamma) = K_{1a}^-(u,\alpha,\beta,\gamma, J_\pm \to e^{\mp i\theta})$

Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(K_1^+(u) T_{1-}(u) K_1^-(u) T_{1-}^{-1}(-u)) \quad T_{1-}(u) = L_{1a_{N+1}}(u) \dots L_{1a_i}(u) \dots L_{1a_1}(u)$$

Building up the boundary XXZ model

Trivial BYBE solution $K_1^-(u)^{QGI} = diag(e^u, e^{-u})$

General diagonal solution by dressing

$$K_1^-(u,\alpha)^{diag} = T_{1a}(u,\alpha)K_1^{-QGI}T_{1a}^{-1}(-u,\alpha) \propto \begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix} = \begin{pmatrix} \cosh(u+\alpha) & 0 \\ 0 & \cosh(u-\alpha) \end{pmatrix}$$

General nondiagonal solution acting in the quantum space $dimW_a = \infty$

$$K_{1a}^-(u,\alpha,\beta,\gamma) = T_{1a}(u,\beta)K_1^-(u,\alpha)^{diag}T_{1a}^{-1}(-u,\beta) \propto \begin{pmatrix} e^\beta P_+ + e^{-\beta}P_- & -J_- e^{i\gamma}\sinh 2u \\ J_+ e^{-i\gamma}\sinh 2u & e^\beta P_- + e^{-\beta}P_+ \end{pmatrix}$$

scalar nondiagonal solution $K_1^-(u,\alpha,\beta,\gamma) = K_{1a}^-(u,\alpha,\beta,\gamma,J_\pm \to e^{\mp i\theta})$

Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(K_1^+(u)T_{1-}(u)K_1^-(u)T_{1-}^{-1}(-u)) \quad T_{1-}(u) = L_{1a_{N+1}}(u)\dots L_{1a_i}(u)\dots L_{1a_1}(u)$$

Hamiltonian: with $K_1^-(u,\alpha_-,\beta_-,\gamma_-)$ and $K_1^+(-u-\eta,\alpha_+,\beta_+,\gamma_+)$

$$\mathcal{H} \propto t'(0) = \frac{1}{2}\sum_{n=1}^{N-1}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + \cosh\eta\,\sigma_n^x\sigma_{n+1}^x) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + (1 \leftrightarrow N)$$

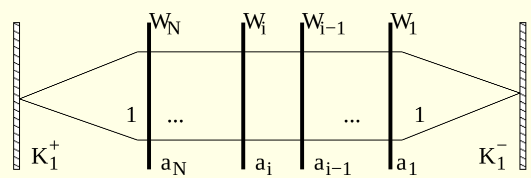
Equivalence in open spin chains: spin order is spectrally irrelevant

Equivalence in open spin chains: spin order is spectrally irrelevant

Transfer matrix: generating functional for conserved charges

$$t(u) = \mathrm{Tr}_1(\mathcal{T}_1^+(u)\mathcal{T}_1^-(u)) \quad ; \qquad \mathcal{T}_1^+(u) = K_1^+(u)\,, \quad \mathcal{T}_1^-(u) = T_{1a}(u)K_1^-(u)T_{1a}^{-1}(-u))$$

$$T_{1-}(u) = L_{1a_N}^N(u)\dots L_{1a_{i+1}}^{i+1}(u)L_{1a_i}^i(u)L_{1a_{i-1}}^{i-1}(u)L_{1a_{i-2}}^{i-2}(u)\dots L_{1a_1}^1(u)$$



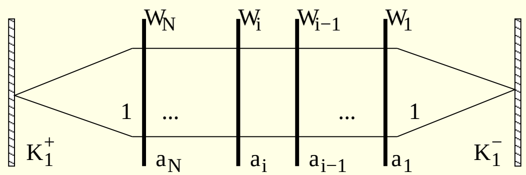
$$K_{1a}^-(u) = T_{1a}(u)K_1^-(u)T_{1a}^{-1}(-u) \quad ; \qquad L_{1a_1}^1 = T_{1a}(u)$$

Equivalence in open spin chains: spin order is spectrally irrelevant

Transfer matrix: generating functional for conserved charges

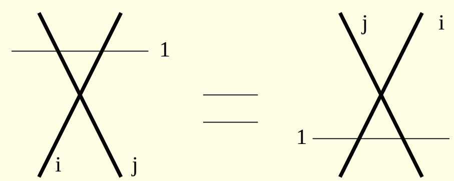
$$t(u) = \mathrm{Tr}_1(\mathcal{T}_1^+(u)\mathcal{T}_1^-(u)) \quad ; \qquad \mathcal{T}_1^+(u) = K_1^+(u)\,, \quad \mathcal{T}_1^-(u) = T_{1a}(u)K_1^-(u)T_{1a}^{-1}(-u))$$

$$T_{1-}(u) = L_{1a_N}^N(u)\ldots L_{1a_{i+1}}^{i+1}(u)L_{1a_i}^i(u)L_{1a_{i-1}}^{i-1}(u)L_{1a_{i-2}}^{i-2}(u)\ldots L_{1a_1}^1(u)$$



$$K_{1a}^-(u) = T_{1a}(u)K_1^-(u)T_{1a}^{-1}(-u) \quad ; \qquad L_{1a_1}^1 = T_{1a}(u)$$

SLL equation:

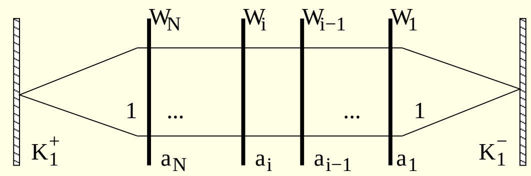


Equivalence in open spin chains: spin order is spectrally irrelevant

Transfer matrix: generating functional for conserved charges

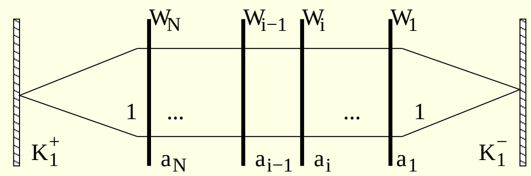
$$t(u) = \mathrm{Tr}_1(\mathcal{T}_1^+(u)\mathcal{T}_1^-(u)) \quad ; \qquad \mathcal{T}_1^+(u) = K_1^+(u)\,, \quad \mathcal{T}_1^-(u) = T_{1a}(u)K_1^-(u)T_{1a}^{-1}(-u))$$

$$T_{1-}(u) = L^N_{1a_N}(u)\dots L^{i+1}_{1a_{i+1}}(u)L^i_{1a_i}(u)L^{i-1}_{1a_{i-1}}(u)L^{i-2}_{1a_{i-2}}(u)\dots L^1_{1a_1}(u)$$



$$K^-_{1a}(u) = T_{1a}(u)K^-_1(u)T^{-1}_{1a}(-u) \quad ; \qquad L^1_{1a_1} = T_{1a}(u)$$

Equivalent chain: $T_{1-}(u) = L^N_{1a_N}(u)\dots L^{i+1}_{1a_{i+1}}(u)L^{i-1}_{1a_{i-1}}(u)L^i_{1a_i}(u)L^{i-2}_{1a_{i-2}}(u)\dots L^1_{1a_1}(u)$

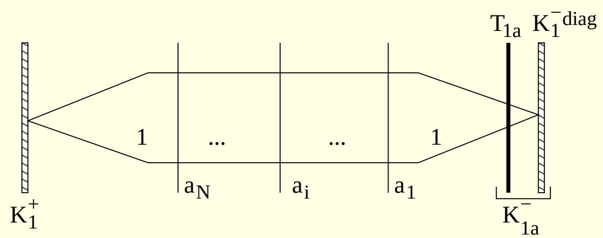


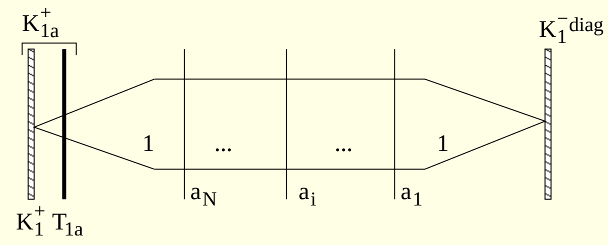
$$K^+_{1a}(u)^{t_1} = T_{1a}(u)^{t_1}K^+_1(u)^{t_1}T^{-1}_{1a}(-u)^{t_1}$$

Equivalence in two boundary XXZ

Equivalence in two boundary XXZ

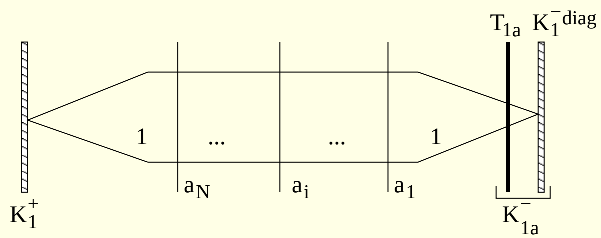
Equivalence between different boundary conditions

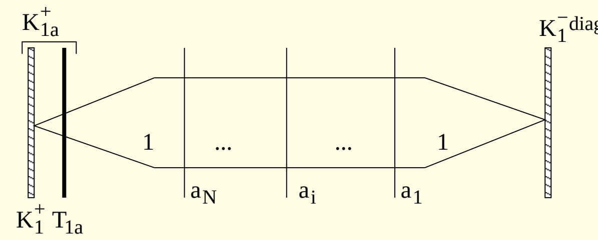




Equivalence in two boundary XXZ

Equivalence between different boundary conditions



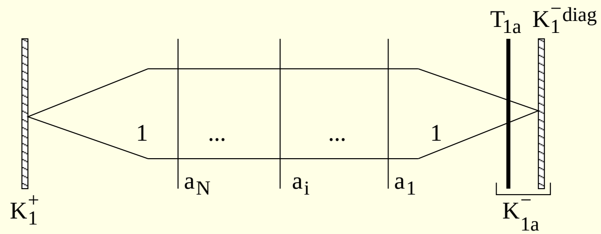


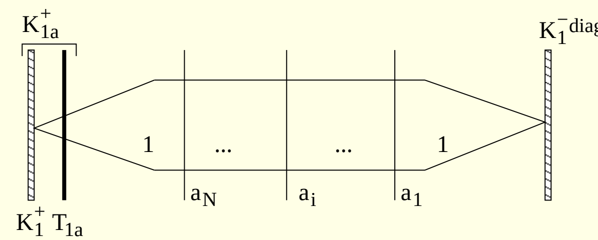
QGI + nondiagonal = diagonal on both sides

$$\left[K_1^+(u) = K_1^-(-u-\eta)^{QGI}\right] + K_{1a}^-(u,\alpha,\beta,\gamma) \quad \equiv \quad \left[K_{1a}^+ = K_1^-(u,\beta)^{diag}\right] + K_1^-(u,\alpha)^{diag}$$

Equivalence in two boundary XXZ

Equivalence between different boundary conditions





QGI + nondiagonal = diagonal on both sides

$$\left[K_1^+(u) = K_1^-(-u-\eta)^{QGI}\right] + K_{1a}^-(u,\alpha,\beta,\gamma) \quad \equiv \quad \left[K_{1a}^+ = K_1^-(u,\beta)^{diag}\right] + K_1^-(u,\alpha)^{diag}$$

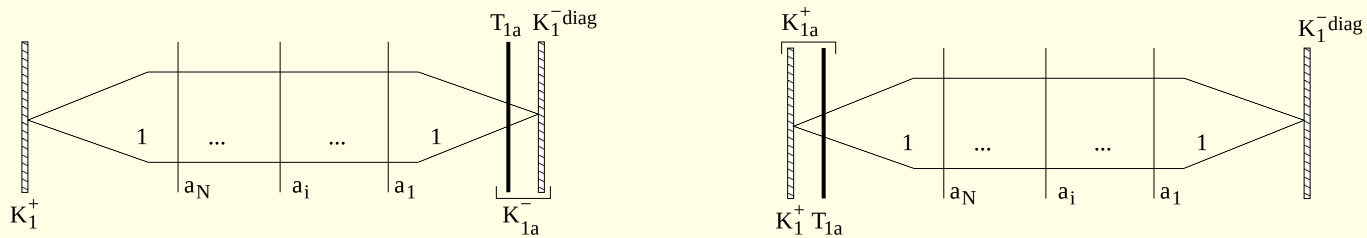
$$diag(e^{-u-\eta}, e^{u+\eta}) + \begin{pmatrix} e^{\beta}P_+ + e^{-\beta}P_- & -J_- e^{i\gamma}\sinh 2u \\ J_+ e^{-i\gamma}\sinh 2u & e^{\beta}P_- + e^{-\beta}P_+ \end{pmatrix} \equiv \begin{pmatrix} \tilde{P}_+ & 0 \\ 0 & \tilde{P}_- \end{pmatrix} + \begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix}$$

$$P_\pm = \cosh(u \pm \alpha) \quad ; \qquad \tilde{P}_\pm = \cosh(-u-\eta \pm \beta)$$

$$K_{1a}^+ = \frac{1}{2}\begin{pmatrix} e^{\beta_- - u - \eta} + e^{-\beta_- + u + \eta} & e^{\mu_2}(e^{-2u-2\eta} - e^{2u+2\eta})q^{J_0} J_- q^{J_0} \\ 0 & e^{\beta_- + u + \eta} + e^{-\beta_- - u - \eta} \end{pmatrix}$$

Equivalence in two boundary XXZ

Equivalence between different boundary conditions



QGI + nondiagonal = diagonal on both sides

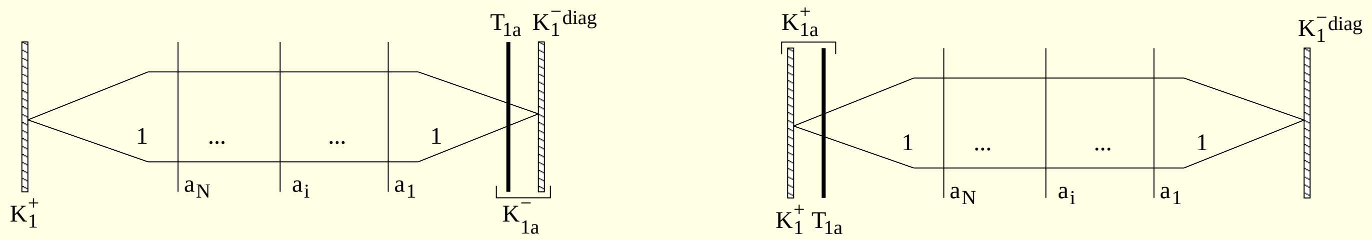
$$\left[K_1^+(u) = K_1^-(-u-\eta)^{QGI}\right] + K_{1a}^-(u,\alpha,\beta,\gamma) \;\equiv\; \left[K_{1a}^+ = K_1^-(u,\beta)^{diag}\right] + K_1^-(u,\alpha)^{diag}$$

nondiagonal on both sides = diagonal + dynamical

$$K_1^+(u,\alpha_+,\beta_+,\gamma_+) + K_{1a}^-(u,\alpha_-,\beta_-,\gamma_-) \;\equiv\; K_1^-(u,\alpha_-)^{diag} + K_{1a}^+(u)^{t_1} = T_{1a}(u)^{t_1} K_1^+(u)^{t_1} T_{1a}^{-1}(-u)^{t_1}$$

Equivalence in two boundary XXZ

Equivalence between different boundary conditions



QGI + nondiagonal = diagonal on both sides

$$\left[K_1^+(u) = K_1^-(-u-\eta)^{QGI}\right] + K_{1a}^-(u,\alpha,\beta,\gamma) \quad \equiv \quad \left[K_{1a}^+ = K_1^-(u,\beta)^{diag}\right] + K_1^-(u,\alpha)^{diag}$$

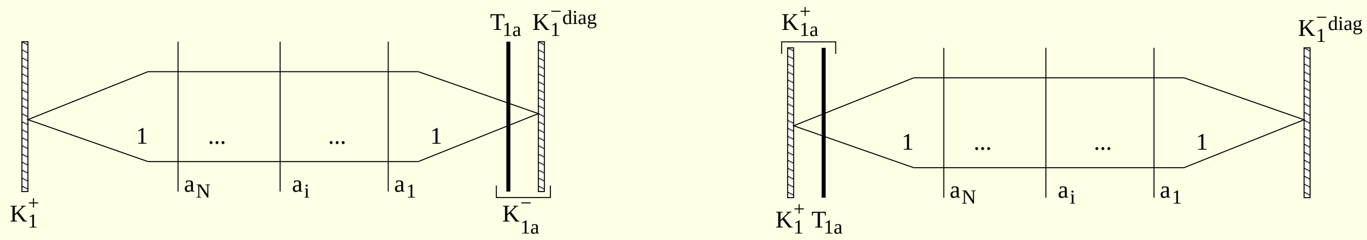
nondiagonal on both sides = diagonal + dynamical

$$K_1^+(u,\alpha_+,\beta_+,\gamma_+) + K_{1a}^-(u,\alpha_-,\beta_-,\gamma_-) \quad \equiv \quad K_1^-(u,\alpha_-)^{diag} + K_{1a}^+(u)^{t_1} = T_{1a}(u)^{t_1} K_1^+(u)^{t_1} T_{1a}^{-1}(-u)^{t_1}$$

$$K_{1a}^+ = \begin{pmatrix} P_{+a}^+ & q^{-2J_0} e^{-\mu_- - \eta} Q_{+a}^+ \\ q^{2J_0} e^{\mu_- - \eta} Q_{-a}^+ & P_{-a}^+ \end{pmatrix} \qquad \begin{aligned} P_{\pm a}^+ &= \sum_{\epsilon=\pm} (e^{\pm\epsilon\beta_-} P_\epsilon^+ \mp e^{\pm\epsilon(u\pm\mu_-)} Q_\epsilon^+ J_{-\epsilon}) \\ Q_{\pm a}^+ &= \sum_{\epsilon=\pm} \epsilon (e^{\pm\epsilon(u+\eta)} P_\epsilon^+ \pm e^{\pm\epsilon(\beta_-+\eta\pm\mu_-)} Q_\epsilon^+ J_{-\epsilon}) J_\pm \end{aligned}$$

Equivalence in two boundary XXZ

Equivalence between different boundary conditions



QGI + nondiagonal = diagonal on both sides

$$\left[K_1^+(u) = K_1^-(-u-\eta)^{QGI}\right] + K_{1a}^-(u,\alpha,\beta,\gamma) \equiv \left[K_{1a}^+ = K_1^-(u,\beta)^{diag}\right] + K_1^-(u,\alpha)^{diag}$$

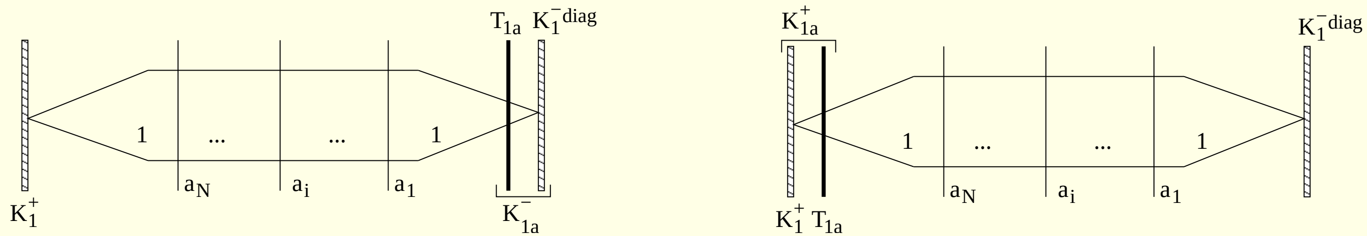
nondiagonal on both sides = diagonal + dynamical

$$K_1^+(u,\alpha_+,\beta_+,\gamma_+) + K_{1a}^-(u,\alpha_-,\beta_-,\gamma_-) \equiv K_1^-(u,\alpha_-)^{diag} + K_{1a}^+(u)^{t_1} = T_{1a}(u)^{t_1} K_1^+(u)^{t_1} T_{1a}^{-1}(-u)^{t_1}$$

one constraint $\beta_+ \mp (\alpha_+ - \gamma_+) = \beta_- \mp (\alpha_- - \gamma_-) + \eta$ dynamical = upper/lower triangular

Equivalence in two boundary XXZ

Equivalence between different boundary conditions



QGI + nondiagonal = diagonal on both sides

$$\left[K_1^+(u) = K_1^-(-u-\eta)^{QGI}\right] + K_{1a}^-(u,\alpha,\beta,\gamma) \quad \equiv \quad \left[K_{1a}^+ = K_1^-(u,\beta)^{diag}\right] + K_1^-(u,\alpha)^{diag}$$

nondiagonal on both sides = diagonal + dynamical

$$K_1^+(u,\alpha_+,\beta_+,\gamma_+) + K_{1a}^-(u,\alpha_-,\beta_-,\gamma_-) \quad \equiv \quad K_1^-(u,\alpha_-)^{diag} + K_{1a}^+(u)^{t_1} = T_{1a}(u)^{t_1} K_1^+(u)^{t_1} T_{1a}^{-1}(-u)^{t_1}$$

one constraint $\beta_+ \mp (\alpha_+ - \gamma_+) = \beta_- \mp (\alpha_- - \gamma_-) + \eta$ dynamical = upper/lower triangular

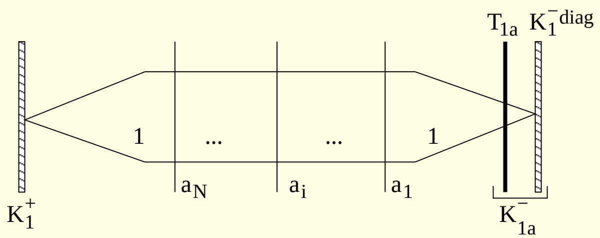
two constraints $\beta_+ = \beta_- + \eta \quad ; \qquad \alpha_+ - \gamma_+ = \alpha_- - \gamma_-$ dynamical = diagonal

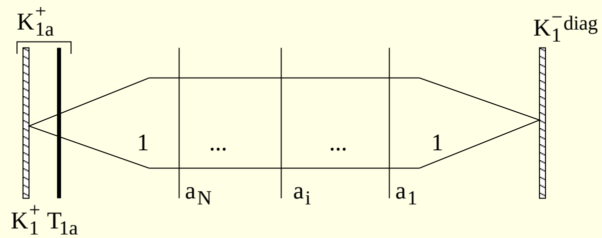
$$K_1^+(u,\alpha_+,\beta_+,\gamma_+) + K_{1a}^-(u,\alpha_-,\beta_-,\gamma_-) \quad \equiv \quad K_1^+(u,\alpha_+)^{diag} + K_1^-(u,\alpha_-)^{diag}$$

Conclusions

Conclusions

Dressing with a defect is a useful tool to calculate K matrices

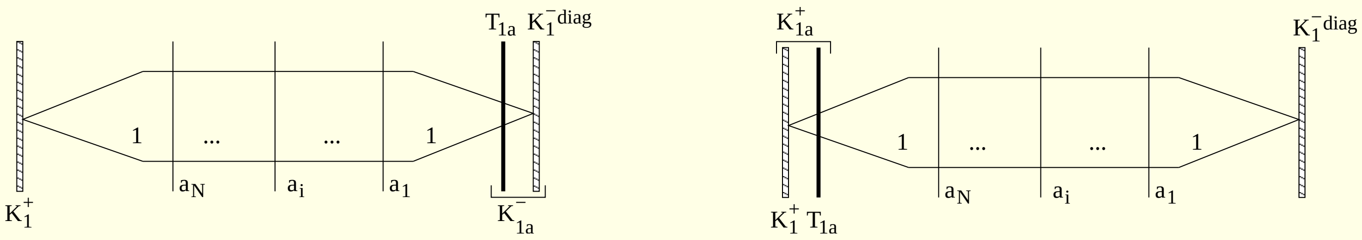




Equivalence between different boundary conditions by moving the dressing defect

Conclusions

Dressing with a defect is a useful tool to calculate K matrices

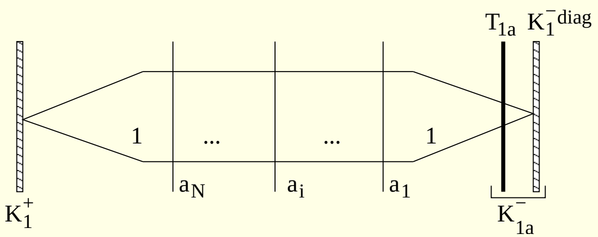


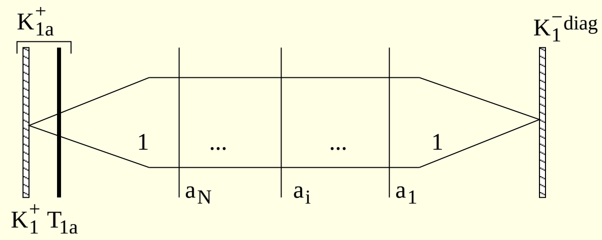
Equivalence between different boundary conditions by moving the dressing defect

No constraint $\longrightarrow$ no BA, dynamical BC. Solve the defect in the closed case first.

Conclusions

Dressing with a defect is a useful tool to calculate K matrices





Equivalence between different boundary conditions by moving the dressing defect

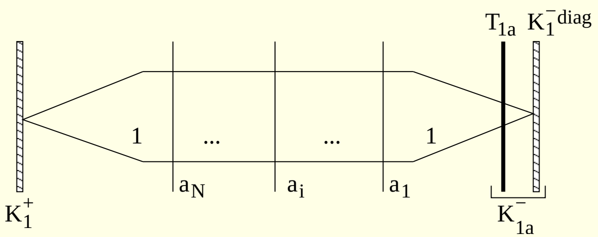
No constraint $\longrightarrow$ no BA, dynamical BC. Solve the defect in the closed case first.

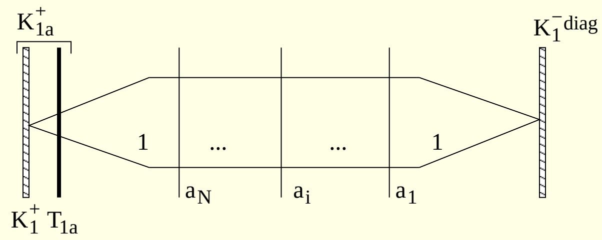
$$\mathcal{H} \;=\; \frac{1}{2}\sum_{n=1}^{N-1}(bulk) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + \gamma_N^z\sigma_N^z + \gamma_N^x\sigma_N^x + \gamma_N^y\sigma_N^y$$

$$\gamma_{1,N}^z = \pm\frac{1}{2}\sinh\eta\coth\widehat{\alpha}_{\mp}\tanh\widehat{\beta}_{\mp}\,; \quad \gamma_{1,N}^x \propto \cosh\widehat{\theta}_{\mp}\,; \quad \gamma_{1,N}^y \propto i\sinh\widehat{\theta}_{\mp}$$

Conclusions

Dressing with a defect is a useful tool to calculate K matrices





Equivalence between different boundary conditions by moving the dressing defect

No constraint $\longrightarrow$ no BA, dynamical BC. Solve the defect in the closed case first.

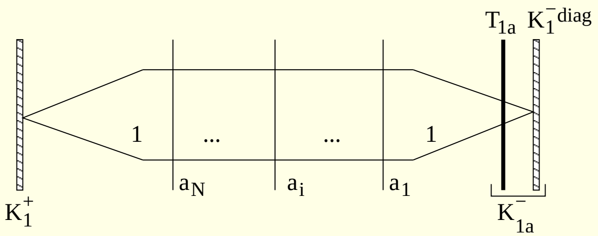
$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N-1}(bulk) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + \gamma_N^z\sigma_N^z + \gamma_N^x\sigma_N^x + \gamma_N^y\sigma_N^y$$

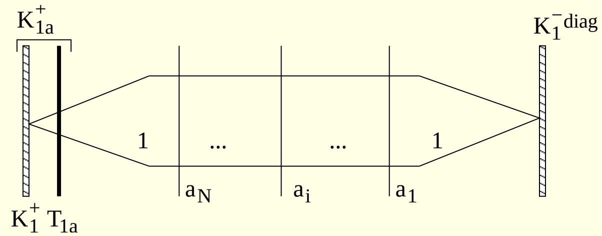
$$\gamma_{1,N}^z = \pm\frac{1}{2}\sinh\eta\coth\widehat{\alpha}_\mp\tanh\widehat{\beta}_\mp\,; \quad \gamma_{1,N}^x \propto \cosh\widehat{\theta}_\mp\,; \quad \gamma_{1,N}^y \propto i\sinh\widehat{\theta}_\mp$$

Global symmetry: rotation in the $x - y$ plane $\longrightarrow$ spectrum depends on $\widehat{\theta}_+ - \widehat{\theta}_-$.

Conclusions

Dressing with a defect is a useful tool to calculate K matrices





Equivalence between different boundary conditions by moving the dressing defect

No constraint $\longrightarrow$ no BA, dynamical BC. Solve the defect in the closed case first.

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N-1}(bulk) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + \gamma_N^z\sigma_N^z + \gamma_N^x\sigma_N^x + \gamma_N^y\sigma_N^y$$

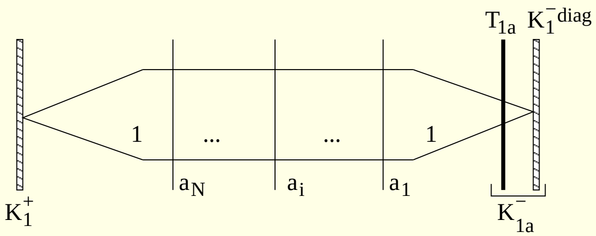
$$\gamma_{1,N}^z = \pm\frac{1}{2}\sinh\eta\coth\widehat{\alpha}_{\mp}\tanh\widehat{\beta}_{\mp}\,; \quad \gamma_{1,N}^x \propto \cosh\widehat{\theta}_{\mp}\,; \quad \gamma_{1,N}^y \propto i\sinh\widehat{\theta}_{\mp}$$

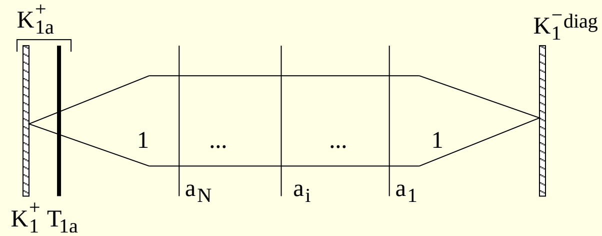
Global symmetry: rotation in the $x - y$ plane $\longrightarrow$ spectrum depends on $\widehat{\theta}_+ - \widehat{\theta}_-$.

This can be implemented by the defect $\Gamma = diag(e^{-\widehat{\theta}_-}, e^{\widehat{\theta}_-})$

Conclusions

Dressing with a defect is a useful tool to calculate K matrices





Equivalence between different boundary conditions by moving the dressing defect

No constraint $\longrightarrow$ no BA, dynamical BC. Solve the defect in the closed case first.

$$\mathcal{H} \;=\; \frac{1}{2}\sum_{n=1}^{N-1}(bulk) + \gamma_1^z\sigma_1^z + \gamma_1^x\sigma_1^x + \gamma_1^y\sigma_1^y + \gamma_N^z\sigma_N^z + \gamma_N^x\sigma_N^x + \gamma_N^y\sigma_N^y$$

$$\gamma_{1,N}^z = \pm\frac{1}{2}\sinh\eta\coth\widehat{\alpha}_\mp\tanh\widehat{\beta}_\mp\,; \quad \gamma_{1,N}^x \propto \cosh\widehat{\theta}_\mp\,; \quad \gamma_{1,N}^y \propto i\sinh\widehat{\theta}_\mp$$

Global symmetry: rotation in the $x-y$ plane $\longrightarrow$ spectrum depends on $\widehat{\theta}_+ - \widehat{\theta}_-$.

This can be implemented by the defect $\Gamma = diag(e^{-\widehat{\theta}_-}, e^{\widehat{\theta}_-})$

More complicated defects perform local gauge transformations