"Recent Advances in Quantum Field and String Theory" in Tbilisi, Georgia, September 26-30, 2011

TBA, NLO Lüscher corrections and double wrapping in twisted AdS/CFT

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non supersym. def. of $\mathcal{N} = 4$ SYM

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2_{\gamma_i} + \overline{\Psi}[\Phi,\Psi]_{\gamma_i}$$

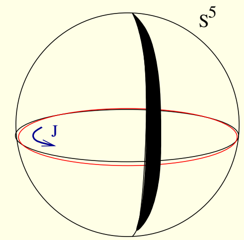
$$[\Phi_i,\Phi_j]_{\gamma_k} = e^{i\epsilon_{ijk}\gamma_k}\Phi_i\Phi_j - e^{-i\epsilon_{ijk}\gamma_k}\Phi_j\Phi_i$$

$$\mathcal{O} = \mathrm{Tr}(Z^J)$$

$$\Delta_{\mathcal{O}} = J + \lambda^J \Delta^{1w}_J + \ldots + \lambda^{2J}\Delta^{2w}_{2J}$$

$\leftrightarrow$

TST deformed AdS



$\equiv$ AdS with twisted BC.

$E(\lambda) = J +$ finite size corr.

Based on: arXiv:1108.4914 TBA, NLO Luscher correction, and double wrapping in twisted AdS/CFT, Changrim Ahn, Zoltan Bajnok, Diego Bombardelli, Rafael I. Nepomechie, AdS/CFT correspondence: (Maldacena 1997)

$\mathcal{N}=4$ D=4 $SU(N)$ SYM

$$A_\mu \;\nearrow\; \Psi_{1,2,3,4} \;\searrow\; \Phi_{1,2,3,4,5,6} \qquad A_\mu \;\searrow\; \overline{\Psi}_{1,2,3,4} \;\nearrow\; \Phi_{1,2,3,4,5,6}$$

$$\frac{2}{g^2_{YM}}\int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2-\frac{1}{2}(D\Phi)^2+i\overline{\Psi}\not{D}\Psi+V\right]$$

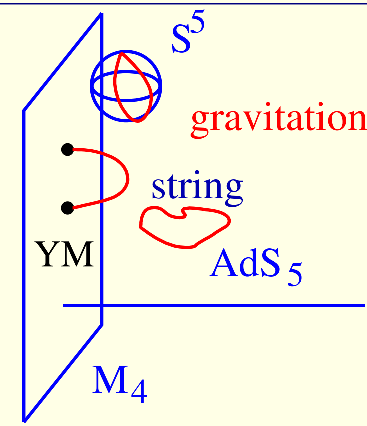
$$V(\Phi,\Psi)=\frac{1}{4}[\Phi,\Phi]^2+\overline{\Psi}[\Phi,\Psi]$$

$\beta=0$ superconformal $\frac{PSU(2,2|4)}{SO(5)\times SO(1,4)}$

gaugeinvariants: $\mathcal{O}=\mathrm{Tr}(\Phi^J)$

$\equiv$

II_B superstring on $AdS_5\times S^5$



$\sum_1^6 Y_i^2=R^2 \qquad -++++-=-R^2$

$$\frac{R^2}{\alpha'}\int\frac{d\tau d\sigma}{4\pi}\left(\partial_a X^M\partial^a X_M+\partial_a Y^M\partial^a Y_M\right)+\dots$$

AdS/CFT correspondence: (Maldacena 1997)

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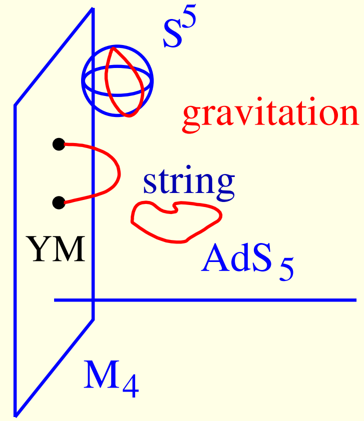

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Dictionary

$\lambda=g_{YM}^2 N$, $N\to\infty$ planar limit

$$\langle\mathcal{O}_n(x)\mathcal{O}_m(0)\rangle=\frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim $\Delta(\lambda)$

$$\Delta(\lambda)=\Delta(0)+\lambda\Delta_1+\lambda^2\Delta_2+\dots$$

weak$\leftrightarrow$strong

$\Downarrow$

Couplings: $\sqrt{\lambda}=\frac{R^2}{\alpha'}$, $g_s=\frac{\lambda}{N}\to 0$

2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda)=E(\infty)+\frac{E_1}{\sqrt{\lambda}}+\frac{E_2}{\lambda}+\dots$$

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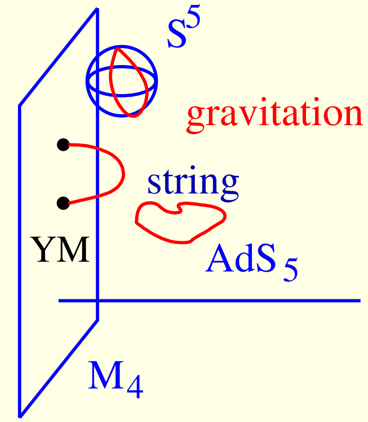

$$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$$

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2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \ldots$$

2D integrable QFT

spectrum: $Q = 1, 2, \ldots, \infty$ dispersion: $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$

Exact scattering matrix: $S_{Q_1 Q_2}(p_1, p_2, \lambda)$

AdS/CFT correspondence in every day life

supersymmetric BPS operators

$$V(\Phi,\Psi)=\frac{1}{4}[\Phi,\Phi]^2+\overline{\Psi}[\Phi,\Psi]$$

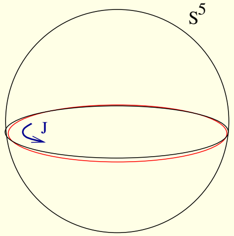
$$Z=\Phi_1+i\Phi_2, X=\Phi_3+i\Phi_4$$

$$\mathcal{O}_{BPS}=\mathrm{Tr}(Z^J)\leftrightarrow|\uparrow\uparrow\ldots\uparrow\rangle$$

$$\Delta_{BPS}=J$$

weak↔strong

BPS string configuration



$$E_{BPS}(\lambda)=J$$

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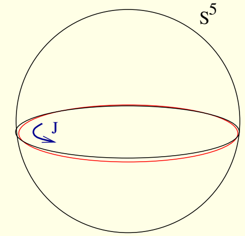

$$E_{BPS}(\lambda) = J$$

2D integrable QFT

supersymmetric groundstate $E_0(J) = \Delta(\lambda) - J = 0$

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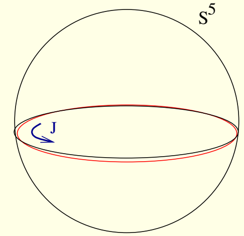


$$E_{BPS}(\lambda) = J$$

2D integrable QFT

supersymmetric groundstate $E_0(J) = \Delta(\lambda) - J = 0$

Nontrivial anomalous dimension

supersymmetric theory: Excited state

$$\mathcal{O}_K = \mathrm{Tr}(ZXZX + \ldots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + .$$

nonsupersymmetric theory: groundstate

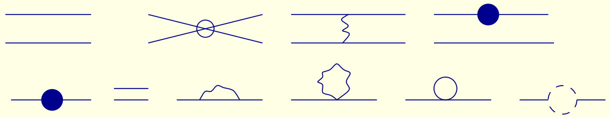
$$[\Phi_i,\Phi_j]_{\gamma_k} = e^{i\epsilon_{ijk}\gamma_k}\Phi_i\Phi_j - e^{-i\epsilon_{ijk}\gamma_k}\Phi_j\Phi_i$$

Supersymmetric theory: excited state - Konishi operator

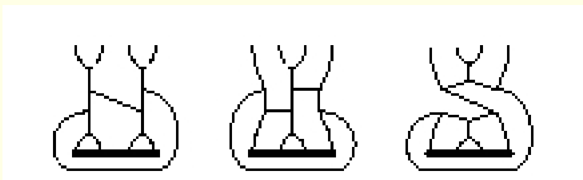
nonsupersymmetric operator: Konishi

$\mathcal{O}_K = \text{Tr}(ZXZX + ...) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + .$

operator mixing



$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \ldots + \lambda^4\Delta_4 +$

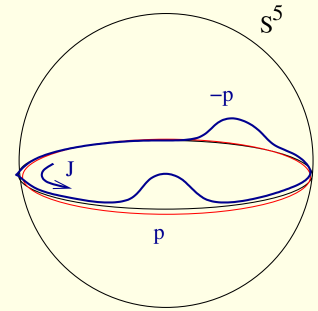


[Fiamberti ..'08]

$\Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$

$\equiv$

string configuration



moving bumps (sine-Gordon) [Hofman .. '07]

finite size correction [Arutyunov .. '07]

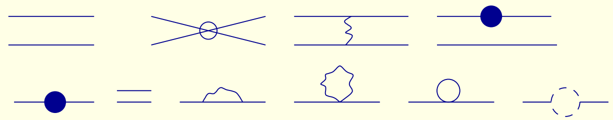
string action=saddle point+loop corr.

Supersymmetric theory: excited state - Konishi operator

nonsupersymmetric operator: Konishi

$\mathcal{O}_K = \mathrm{Tr}(ZXZX + ...) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + .$

operator mixing



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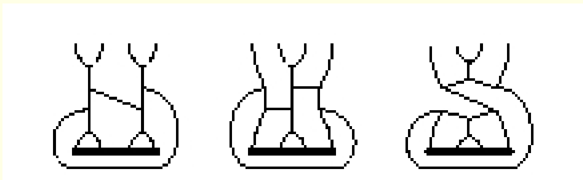

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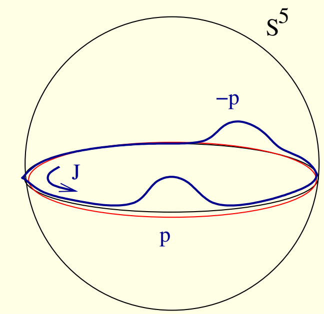
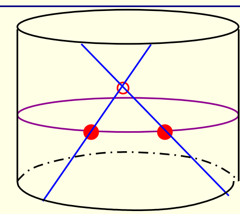


moving bumps (sine-Gordon) [Hofman .. '07]

finite size correction [Arutyunov .. '07]

string action=saddle point+loop corr.

two particle state



$$E = E_{BA} + E_{FSC}$$

Bethe Ansatz: $e^{ipJ}S(p,-p) = 1$

$$E_{BA} = 2E(p,\lambda) = 2\sqrt{1 + \frac{\lambda}{\pi^2}(\sin\frac{p}{2})^2}$$

$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$ [Z.B. .. '09]

AdS/CFT: twisted theory

non supersymmetric theory [Frolov .. '05]

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2_{\gamma_i} + \overline{\Psi}[\Phi,\Psi]_{\gamma_i}$$

$$[\Phi_i,\Phi_j]_{\gamma_k} = e^{i\epsilon_{ijk}\gamma_k}\Phi_i\Phi_j - e^{-i\epsilon_{ijk}\gamma_k}\Phi_j\Phi_i$$

$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

$$\mathcal{O} = \mathrm{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow\ldots\uparrow\rangle$$

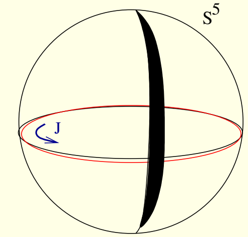
$$\Delta_{\mathcal{O}} = J + \Delta_{\text{wrap.}}$$

$$= J + \lambda^J\Delta_J + \ldots + \lambda^{2J}\Delta_{2J}$$

$\leftrightarrow$

TST deformed AdS

[Frolov '05][Alday .. '06]

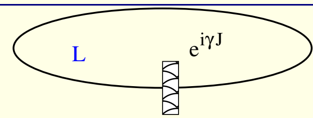


$\equiv$AdS with twisted BC.

$E(\lambda) = J+$finite size corr.

[Arutyunov .. '11]

twisted groundstate

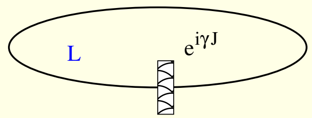


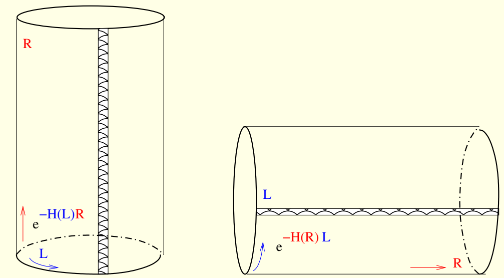
$$E - J = E_{FSC}$$

E_{FSC} =finite size correction!

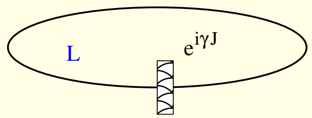
Understand twisted vacuum energy

Plan

Ground-state energy 

the basic idea of TBA 

Plan

Ground-state energy 

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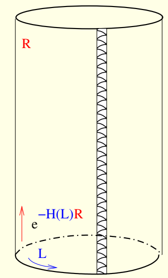
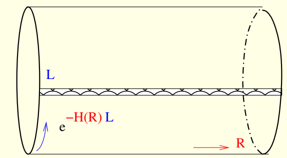
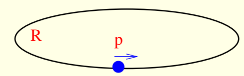
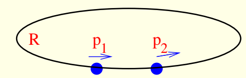




Cluster expansion: LO and NLO Lüscher corrections





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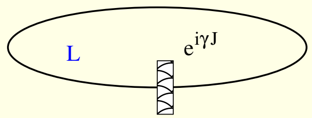
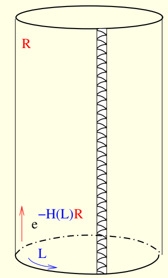
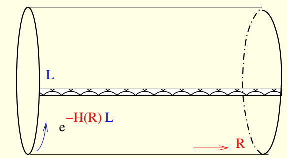
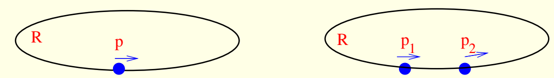
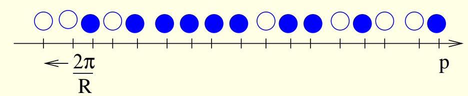
Ground-state energy 

the basic idea of TBA 



Cluster expansion: LO and NLO Lüscher corrections 

Twisted TBA equations, untwisted Y-system 

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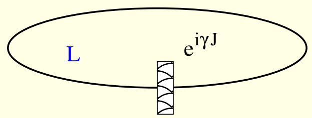
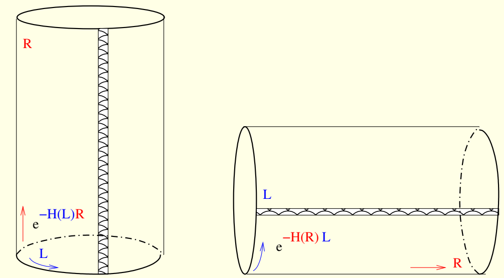
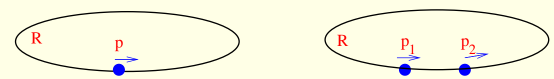
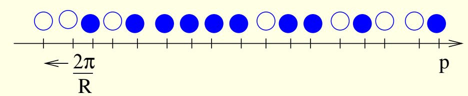
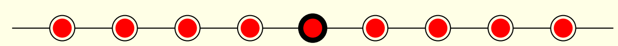
Ground-state energy  the basic idea of TBA 

Cluster expansion: LO and NLO Lüscher corrections 

Twisted TBA equations, untwisted Y-system 

O(4) model: LO and NLO Lüscher and twisted TBA 

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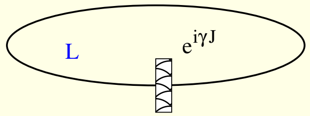
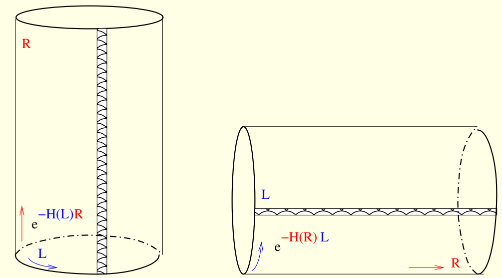
Ground-state energy  the basic idea of TBA



Cluster expansion: LO and NLO Lüscher corrections

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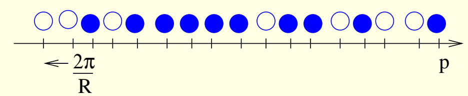
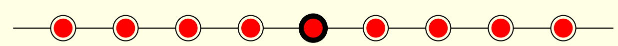
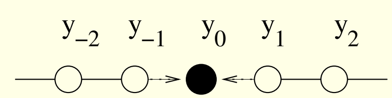


O(4) model: LO and NLO Lüscher and twisted TBA



O(4) model: Asymptotic expansion of TBA vs. Lüscher corrections



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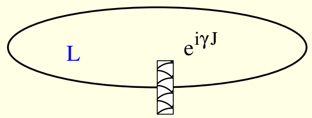
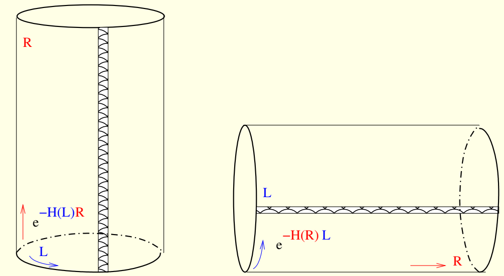
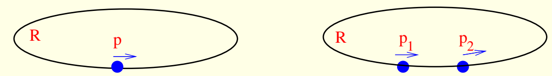
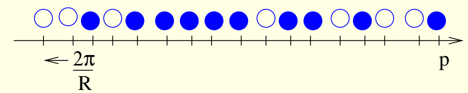
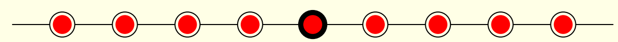
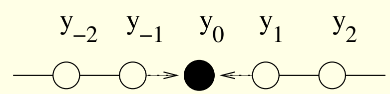
Ground-state energy 

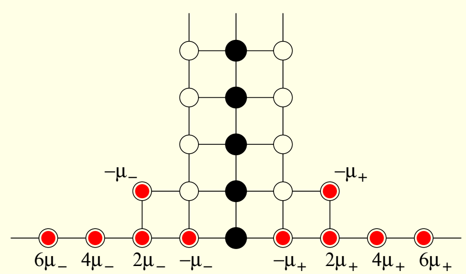
the basic idea of TBA 

Cluster expansion: LO and NLO Lüscher corrections 

Twisted TBA equations, untwisted Y-system 

O(4) model: LO and NLO Lüscher and twisted TBA 

O(4) model: Asymptotic expansion of TBA vs. Lüscher corrections 



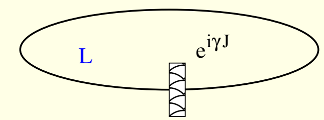
Consequences for AdS

Conclusion, outlook

Ground-state energy in a finite volume with twisted BC

Ground-state energy with twisted BC ($e^{i\gamma J}$)

J conserved charge



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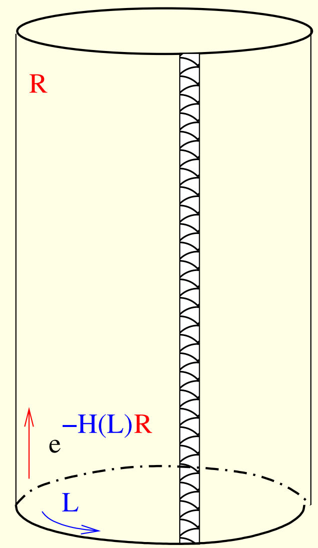
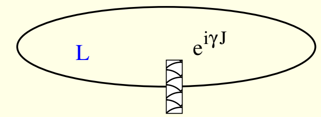
Euclidean twisted partition function:

$$Z^t(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H^t(L)R})$$
$$Z^t(L,R) =_{R\to\infty} e^{-E_0(L)R}(1+e^{-\Delta E\, R})$$





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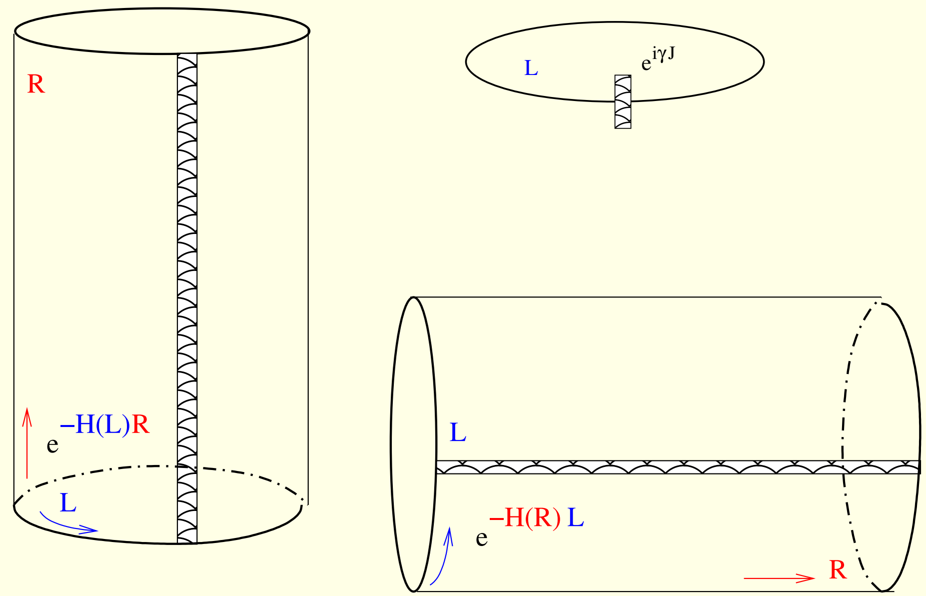

Exchange space and Euclidean time

$$Z^t(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H(R)L}D) =_{R\to\infty} \sum_n e^{-E_n(R)L+i\gamma J_n}$$

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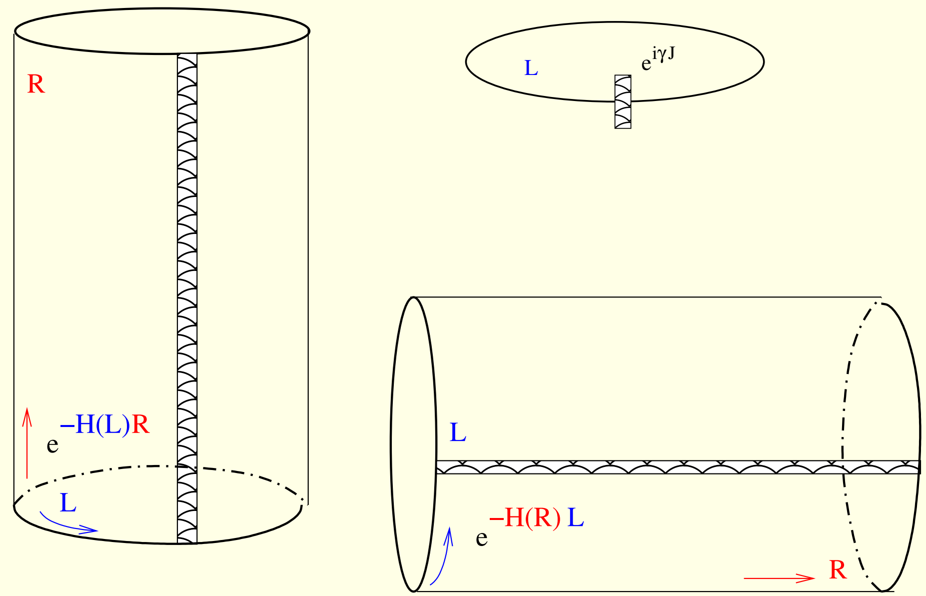

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Large (L) volume (cluster) expansion

$\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J}) = 1 +$ one particle term $+$ two particle term $+$ ...

$$\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J}) = 1 + \sum_{k,\alpha} e^{i\gamma J_\alpha - e(p_k)L} + \sum_{k,l,(\alpha,\beta)} e^{i\gamma J_{(\alpha,\beta)}-(e(p_k)+e(p_l))L} + \dots$$

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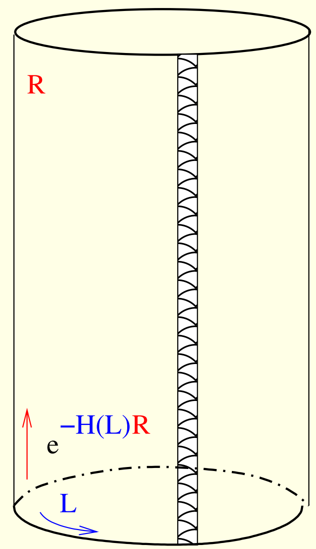

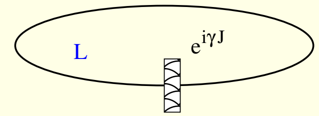

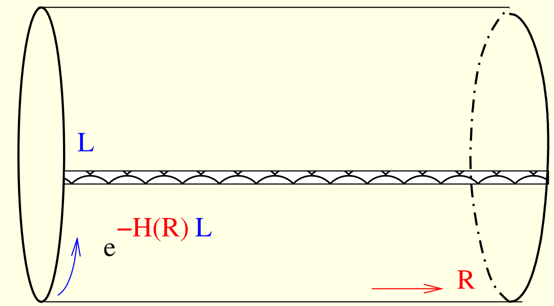

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$$\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J}) = 1 + \sum_{k,\alpha} e^{i\gamma J_\alpha - e(p_k)L} + \sum_{k,l,(\alpha,\beta)} e^{i\gamma J_{(\alpha,\beta)} - (e(p_k)+e(p_l))L} + \dots$$

Groundstate energy:

$$-\lim_{R\to\infty}\frac{1}{R}\log(\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J})) = E_0(L) = \text{LO Lüscher} + \text{NLO Lüscher} + \dots = TBA$$

LO and NLO Lüscher corrections

Large (L) volume (cluster) expansion of the twisted partition function

$\mathrm{Tr}(e^{-H(R)L}e^{i\gamma J}) = 1 + \sum_{k,\alpha} e^{i\gamma J_\alpha - e(p_k)L} + \sum_{k>l,(\alpha,\beta)} e^{i\gamma J_{(\alpha,\beta)} - (e(p_k)+e(p_l))L} + \dots$

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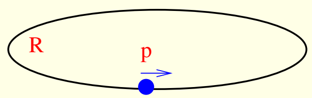
One particle term: 

mom. quant.: $e^{ip_kR} = 1 \to \frac{R}{2\pi}p_k = k \in \mathbb{Z}$

Expanding the log $\log(1+x) = x - \frac{x^2}{2} + \dots$ change $\sum_k \to R\int \frac{dp}{2\pi}$

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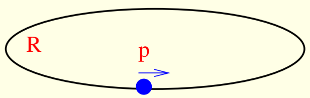
One particle term: 

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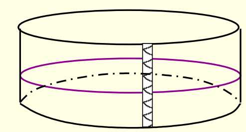

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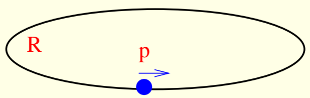
One particle term: 

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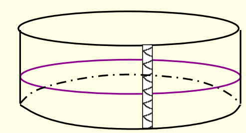
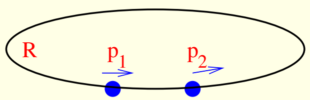


Two particle term: 

$$e^{ip_1R}S(p_1,p_2) = 1 \quad \to \quad p_1R + \delta(1,2) = 2\pi k_1$$
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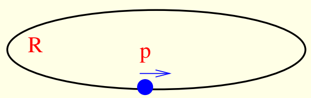

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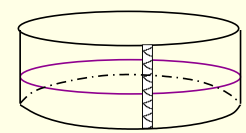

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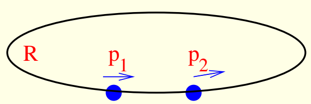

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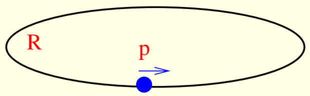
One particle term: 

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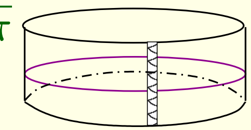

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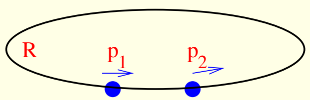
Two particle term: 

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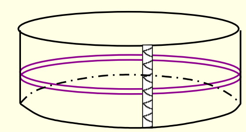
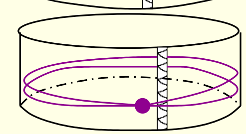
NLO Lüscher correction: $$E_0^{(2,1)}(L) = \frac{1}{2}\mathrm{Tr}_1(e^{i\gamma J})^2\int\frac{dp}{2\pi}e^{-2e(p)L}$$



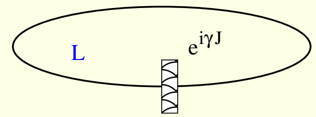
$$E_0^{(2,2)}(L) = \int\frac{dp_1}{2\pi}e^{-e(p_1)L}\int\frac{dp_2}{2\pi}e^{-e(p_2)L}i\partial_{p_1}\mathrm{Tr}_2(e^{i\gamma J}\log S(p_1,p_2))\dots$$



NLO: [Dashen .. '69] Cluster expansion: [Dorey .. '04], [Z.B. .. '05]

Twisted TBA equations

Ground-state energy exactly



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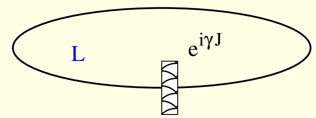
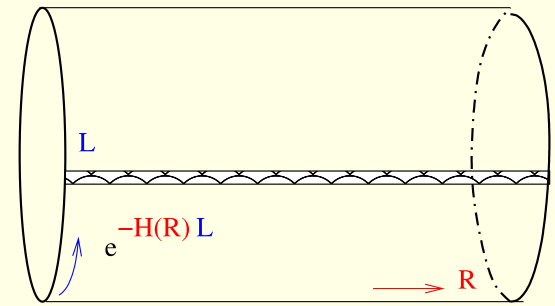


Euclidean twisted partition function, rotated:



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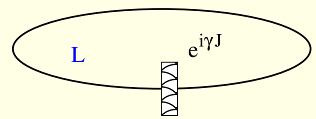

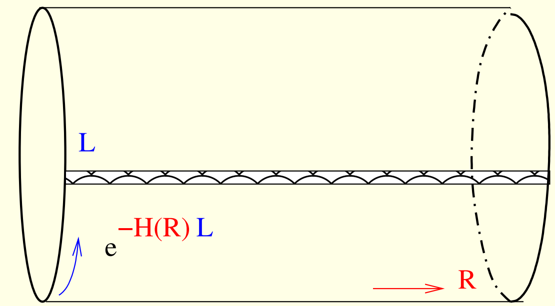

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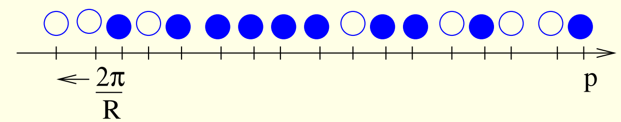
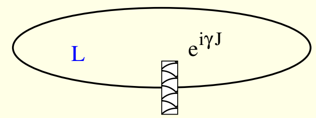
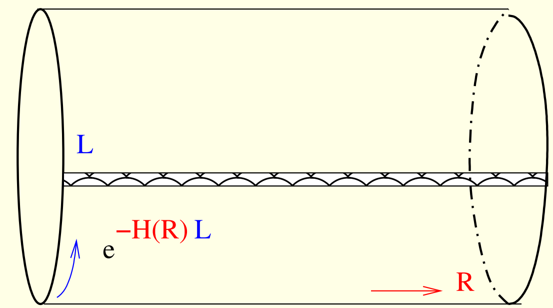


Dominant contribution: finite particle/hole density ρ, ρ_h:

Twisted TBA equations

Ground-state energy exactly





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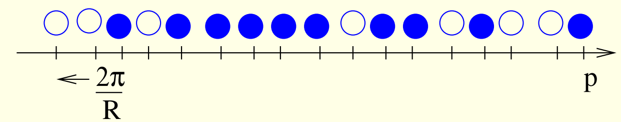


Dominant contribution: finite particle/hole density ρ, ρ_h:

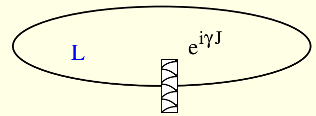
$E_n(R) = \sum_i E(p_i) \to R \int e^Q(p)\rho^Q(p)dp$ $\qquad$ $i\gamma J_n = \mu = R\int \mu^Q(p)\rho^Q(p)dp$

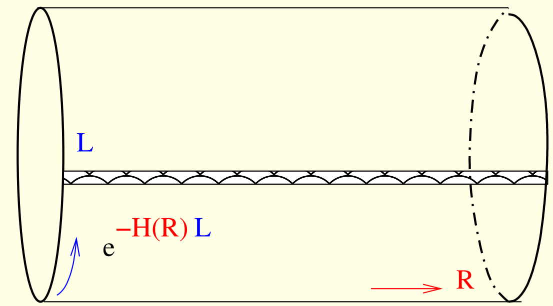
momentum quantization:

$$p_j^{Q_j} R + \sum_k \frac{1}{i} \log S^{Q_j Q_k}(p_j, p_k) = (2n+1)i\pi \quad \longrightarrow R + \int(-id_p \log S^{QQ'}(p,p'))\rho^{Q'}(p')dp' = 2\pi(\rho^Q + \rho_h^Q)$$

Twisted TBA equations

Ground-state energy exactly





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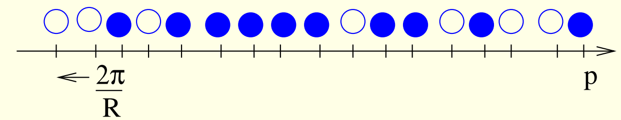


Dominant contribution: finite particle/hole density ρ, ρ_h:

$E_n(R) = \sum_i E(p_i) \to R\int e^Q(p)\rho^Q(p)dp$ $\qquad$ $i\gamma J_n = \mu = R\int \mu^Q(p)\rho^Q(p)dp$

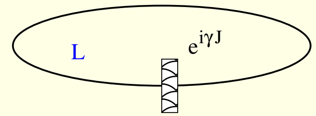
momentum quantization:

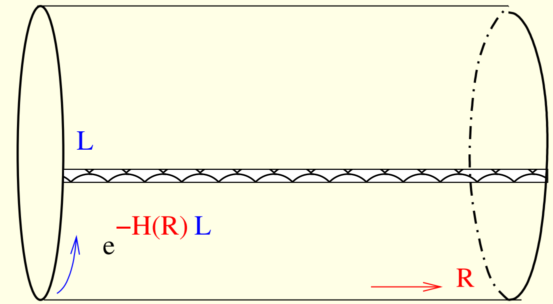
$p_j^{Q_j}R + \sum_k \frac{1}{i}\log S^{Q_jQ_k}(p_j,p_k) = (2n+1)i\pi \quad \longrightarrow R + \int(-id_p \log S^{QQ'}(p,p'))\rho^{Q'}(p')dp' = 2\pi(\rho^Q + \rho_h^Q)$

Partition function: $Z(L,R) = \int d[\rho^Q, \rho_h^Q] e^{-LE+\mu-s}$

entropy factor: $s = \int((\rho^Q+\rho_h^Q)\ln(\rho^Q+\rho_h^Q) - \rho^Q\ln\rho^Q - \rho_h^Q\ln\rho_h^Q)dp$

Twisted TBA equations





Ground-state energy exactly

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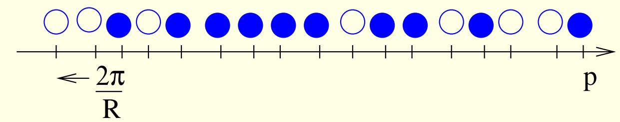

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entropy factor: $s = \int((\rho^Q+\rho_h^Q)\ln(\rho^Q+\rho_h^Q) - \rho^Q\ln\rho^Q - \rho_h^Q\ln\rho_h^Q)dp$

Saddle point for pseudo energy: $\epsilon^Q(p) = \ln\frac{\rho_h^Q(p)}{\rho^Q(p)}$

$$\epsilon^Q(p) + \mu^Q = e^Q(p)L + \int\frac{dp}{2\pi}id_p\log S^{Q'Q}(p',p)\log(1+e^{-\epsilon^{Q'}(p')})$$

Ground state energy exactly: $E_0(L) = -\sum_Q\int\frac{dp}{2\pi}\log(1+e^{-\epsilon^Q(p)})$ [Al.B. Zamolodchikov '90]

O(4) model: LO and NLO Lüscher correction

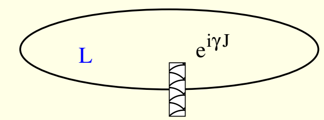
Relativistic theory, one multiplet of mass m: $e(\theta) = m \cosh \pi\theta \quad ; \; p(\theta) = m \sinh \pi\theta$

Factorized scattering, $su(2) \otimes su(2)$ invariance: $(\dot{\uparrow}, \dot{\downarrow}) \otimes (\uparrow, \downarrow)$ [Zamolodchikovs '79]

$$S(\theta) = \frac{S_0^2(\theta)}{(\theta - i)^2} \hat{S}(\theta) \otimes \hat{S}(\theta) \qquad \hat{S}(\theta) = \theta\,\mathbb{I} - i\,\mathbb{P} \qquad S_0(\theta) = i \frac{\Gamma(\frac{1}{2} - \frac{i\theta}{2})\Gamma(\frac{i\theta}{2})}{\Gamma(\frac{1}{2} + \frac{i\theta}{2})\Gamma(-\frac{i\theta}{2})}$$

Twist $e^{i\gamma J} = e^{i\gamma_- J_0 \otimes \mathbb{I} + i\gamma_+ \mathbb{I} \otimes J_0} = e^{i\gamma_- J_0} \otimes e^{i\gamma_+ J_0}$

$$= \mathrm{diag}(\dot{q}, \dot{q}^{-1}) \otimes \mathrm{diag}(q, q^{-1}) \quad ; \qquad q = e^{i\gamma}$$



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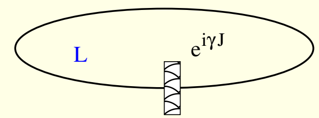
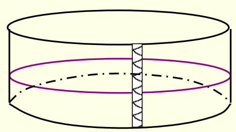


$$[n]_q = \frac{q^n - q^{-n}}{q - q^{-1}}$$

LO Lüscher correction:  $= -[2]_q[2]_{\dot q}m\int\frac{d\theta}{2}\cosh\pi\theta\,e^{-mL\cosh\pi\theta}$

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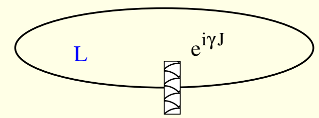

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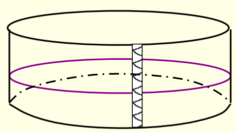
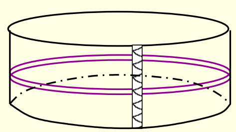
LO Lüscher correction:  $= -[2]_q [2]_{\dot{q}} m \int \frac{d\theta}{2} \cosh \pi\theta \, e^{-mL \cosh \pi\theta}$

NLO Lüscher correction:  $= \frac{1}{2} [2]_q^2 [2]_{\dot{q}}^2 m \int \frac{d\theta}{2} \cosh \pi\theta \, e^{-2mL \cosh \pi\theta}$

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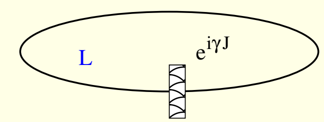

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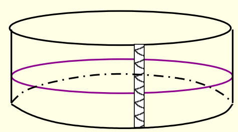
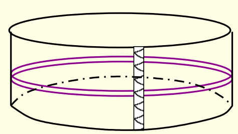
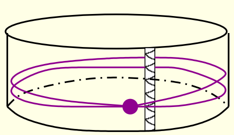
LO Lüscher correction:  $= -[2]_q [2]_{\dot{q}} m \int \frac{d\theta}{2} \cosh \pi\theta \, e^{-mL \cosh \pi\theta}$

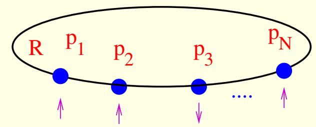
NLO Lüscher correction:  $= \frac{1}{2} [2]_q^2 [2]_{\dot{q}}^2 m \int \frac{d\theta}{2} \cosh \pi\theta \, e^{-2mL \cosh \pi\theta}$

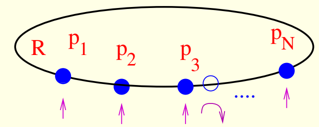
 $= -m \int \frac{d\theta_1}{2} \cosh \pi\theta_1 \, e^{-mL \cosh \pi\theta_1} \int \frac{d\theta_2}{2} \cosh \pi\theta_2 \, e^{-mL \cosh \pi\theta_2} \text{Tr}_2(\text{D}\log S)$

$$\text{Tr}_2(\text{D}\log \text{S}) = \text{Tr}_2(e^{i\gamma J}(-i\partial_\theta) \log S) = -i [2]_q^2 [2]_{\dot{q}}^2 \partial_\theta \log S_0^2(\theta) - i \left([2]_q^2 + [2]_{\dot{q}}^2 \right) \partial_\theta \log \frac{\theta + i}{\theta - i}$$

O(4) model: twisted TBA and Y-system

Particle content in the thermodynamic limit:





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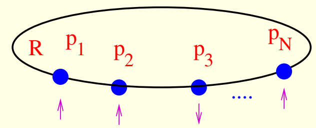

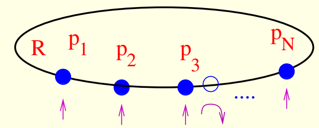

0. massive particle: $|\dot{\downarrow}\rangle \otimes |\downarrow\rangle$ $\begin{array}{c} e_0(\theta) = m\cosh\pi\theta \\ p_0(\theta) = m\sinh\pi\theta \end{array}$ $S_{00}(\theta) = S_0^2(\theta)$ $\mu_0 = -i(\gamma_- + \gamma_+)$

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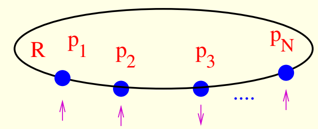

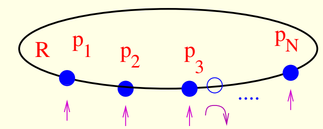

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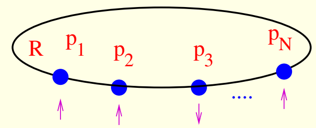

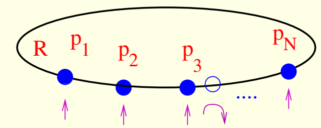

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$\pm$N. magnon boundstate: $\begin{matrix} e_N(\theta) = 0 \\ p_N(\theta) = 0 \end{matrix}$ $\quad S_{0N}(\theta) = \prod_j S_{01} = \frac{\theta + \frac{iN}{2}}{\theta - \frac{iN}{2}}$; $S_{NN'}(u) \;\; \mu_N = i2N\gamma_\pm$

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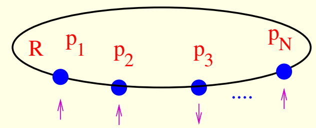

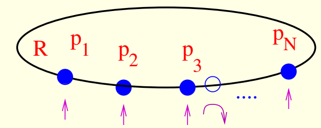

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Twisted TBA equations: [Zamolodchikovs'] $\qquad K_{nm}(u) = \frac{1}{2\pi i}\partial_u \log S_{nm}(u)$

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$$\epsilon_M + \mu_M = -\log(1 + e^{-\epsilon_0}) \star K_{0M} + \sum_{M' \neq 0} \log(1 + e^{-\epsilon_{M'}}) \star K_{M'M}$$

O(4) model: twisted TBA and Y-system

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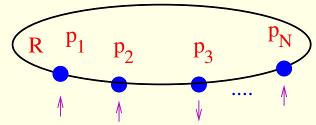

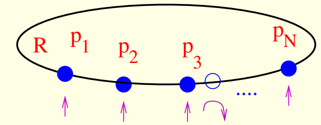

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Groundstate energy: $\quad E_0(L) = -\frac{m}{2}\int d\theta \cosh\pi\theta \, \log(1 + Y_0)$; $\quad Y_0 = e^{-\epsilon_0}$

O(4) model: twisted TBA and Y-system

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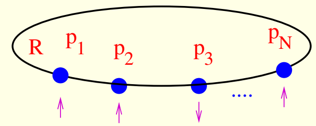

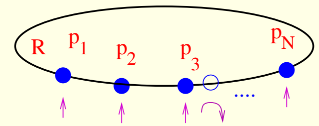

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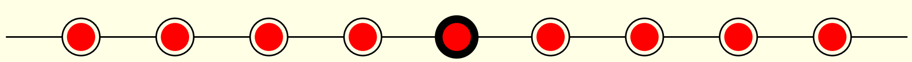
Groundstate energy: $\quad E_0(L) = -\frac{m}{2}\int d\theta \cosh \pi\theta \, \log(1+Y_0)$; $\quad Y_0 = e^{-\epsilon_0}$

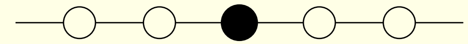
Universal TBA equations and $Y_{M\neq 0} = e^{\epsilon_M}$ Y-system: $\frac{1}{2}(\mu_{N-1} + \mu_{N+1}) - \mu_N = 0$

$$\log Y_n + \delta_{n,0} Lm \cosh \pi\theta = \log\Big((1+Y_{n-1})(1+Y_{n+1})\Big) \star s \quad s(\theta) = \frac{1}{2\cosh \pi\theta}$$

$\lim_{M\to\infty} \log Y_M = 2iM\gamma_+$



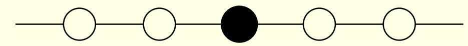
O(4) model: TBA asymptotic expansion

Groundstate energy: $E_0(L) = -\frac{m}{2}\int d\theta \cosh\pi\theta \, \log(1+Y_0)$ 

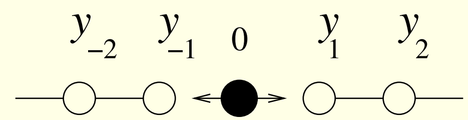
$\log Y_n = -\delta_{n,0} Lm \cosh\pi\theta + \log\Big((1+Y_{n-1})(1+Y_{n+1})\Big) \star s$; $\frac{\log Y_M}{M} \to 2i\gamma_+$

O(4) model: TBA asymptotic expansion

Groundstate energy: $E_0(L) = -\frac{m}{2}\int d\theta \cosh\pi\theta \, \log(1+Y_0)$



$\log Y_n = -\delta_{n,0} Lm \cosh\pi\theta + \log\Big((1+Y_{n-1})(1+Y_{n+1})\Big) \star s$; $\frac{\log Y_M}{M} \to 2i\gamma_+$

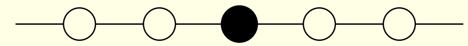


Asymptotic expansion: $Y_M = \mathcal{Y}_M(1+y_M) + \ldots$

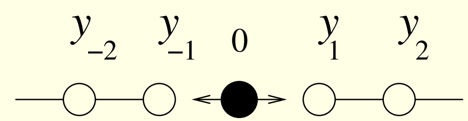
$(\mathcal{Y}_M)^2 = (1+\mathcal{Y}_{M-1})(1+\mathcal{Y}_{M+1}) \longrightarrow \mathcal{Y}_M = [M]_q[M+2]_q$; $\mathcal{Y}_{-M} = \mathcal{Y}_M(q \leftrightarrow \dot{q})$

O(4) model: TBA asymptotic expansion

Groundstate energy: $E_0(L) = -\frac{m}{2}\int d\theta \cosh \pi\theta \, \log(1+Y_0)$



$\log Y_n = -\delta_{n,0} Lm \cosh \pi\theta + \log\Big((1+Y_{n-1})(1+Y_{n+1})\Big) \star s$; $\frac{\log Y_M}{M} \to 2i\gamma_+$



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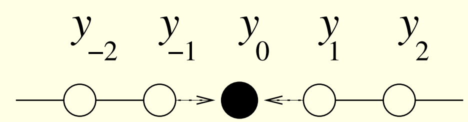
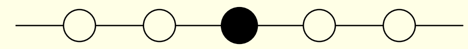


Energy: $\log(1+\mathcal{Y}_0) = \mathcal{Y}_0 - \frac{1}{2}\mathcal{Y}_0^2 \to$ LO and NL0 $E_0^{(1)}, E_0^{(2,1)}$

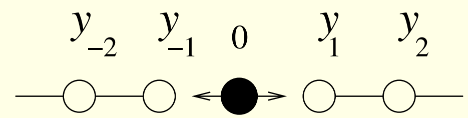
LO contribution $\mathcal{Y}_0 = \sqrt{(1+\mathcal{Y}_1)(1+\mathcal{Y}_{-1})} e^{-mL\cosh\pi\theta} = [2]_q[2]_{\dot{q}} e^{-mL\cosh\pi\theta}$

O(4) model: TBA asymptotic expansion

Groundstate energy: $E_0(L) = -\frac{m}{2}\int d\theta \cosh\pi\theta \, \log(1+Y_0)$

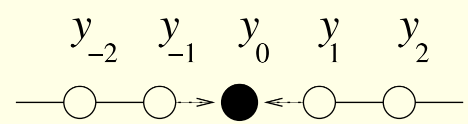


$\log Y_n = -\delta_{n,0} Lm \cosh\pi\theta + \log\Big((1+Y_{n-1})(1+Y_{n+1})\Big)\star s$; $\frac{\log Y_M}{M} \to 2i\gamma_+$



Asymptotic expansion: $Y_M = \mathcal{Y}_M(1+y_M)+\dots$

$(\mathcal{Y}_M)^2 = (1+\mathcal{Y}_{M-1})(1+\mathcal{Y}_{M+1}) \longrightarrow \mathcal{Y}_M = [M]_q[M+2]_q$; $\mathcal{Y}_{-M} = \mathcal{Y}_M(q \leftrightarrow \dot{q})$



Energy: $\log(1+\mathcal{Y}_0) = \mathcal{Y}_0 - \frac{1}{2}\mathcal{Y}_0^2 \to$ LO and NLO $E_0^{(1)}, E_0^{(2,1)}$

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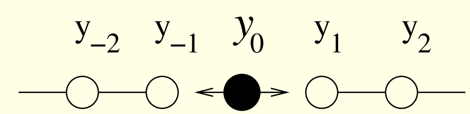
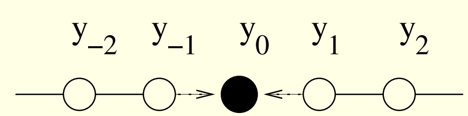


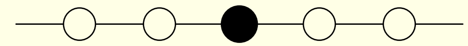
NLO: lin. diff. eq.: $y_k = s\star\Big[\mathcal{A}_{k+1}y_{k+1} + \mathcal{A}_{k-1}y_{k-1}\Big]$; $\mathcal{A}_k = \frac{1+\mathcal{Y}_k}{\mathcal{Y}_k}$



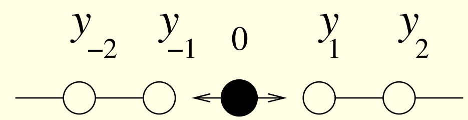
Solution: $\tilde{y}_k = e^{-\frac{|\omega|}{2}k}\frac{[k+1]_q}{[2]_q[k]_q[k+2]_q}([k+2]_q - [k]_q e^{-|\omega|})\tilde{\mathcal{Y}}_0$

O(4) model: TBA asymptotic expansion

Groundstate energy: $E_0(L) = -\frac{m}{2}\int d\theta \cosh \pi\theta \, \log(1+Y_0)$

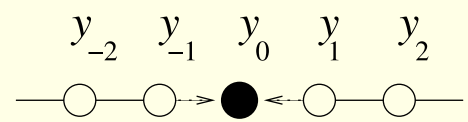


$\log Y_n = -\delta_{n,0} Lm \cosh \pi\theta + \log\Big((1+Y_{n-1})(1+Y_{n+1})\Big) \star s$; $\frac{\log Y_M}{M} \to 2i\gamma_+$



Asymptotic expansion: $Y_M = \mathcal{Y}_M(1+y_M)+\dots$

$(\mathcal{Y}_M)^2 = (1+\mathcal{Y}_{M-1})(1+\mathcal{Y}_{M+1}) \longrightarrow \mathcal{Y}_M = [M]_q[M+2]_q$; $\mathcal{Y}_{-M} = \mathcal{Y}_M(q \leftrightarrow \dot{q})$



Energy: $\log(1+\mathcal{Y}_0) = \mathcal{Y}_0 - \frac{1}{2}\mathcal{Y}_0^2 \to$ LO and NLO $E_0^{(1)}, E_0^{(2,1)}$

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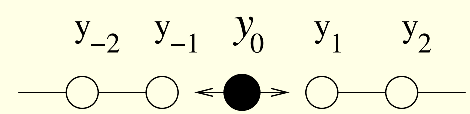
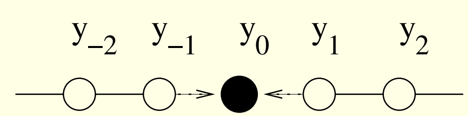


NLO: lin. diff. eq.: $y_k = s\star\Big[\mathcal{A}_{k+1}y_{k+1} + \mathcal{A}_{k-1}y_{k-1}\Big]$; $\mathcal{A}_k = \frac{1+\mathcal{Y}_k}{\mathcal{Y}_k}$



Solution: $\tilde{y}_k = e^{-\frac{|\omega|}{2}k}\frac{[k+1]_q}{[2]_q[k]_q[k+2]_q}([k+2]_q - [k]_q e^{-|\omega|})\tilde{\mathcal{Y}}_0$

NLO contribution: $Y_0 = \mathcal{Y}_0 y_0\,(1 + s\star[\mathcal{A}_1 y_1 + \mathcal{A}_1 y_{-1}])$ Agrees with NLO Lüscher!

Application: twisted AdS/CFT

non supersymmetric theory

$$V(\Phi,\Psi)=\tfrac{1}{4}[\Phi,\Phi]^2_{\gamma_i}+\overline{\Psi}[\Phi,\Psi]_{\gamma_i}$$

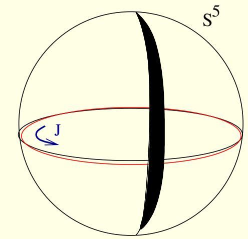
$$[\Phi_i,\Phi_j]_{\gamma_k}=e^{i\epsilon_{ijk}\gamma_k}\Phi_i\Phi_j-e^{-i\epsilon_{ijk}\gamma_k}\Phi_j\Phi_i$$

$$\mathcal{O}=\mathrm{Tr}(Z^J)$$

$$\Delta_{\mathcal{O}}=J+\lambda^J\Delta^{1w}_J+\ldots+\lambda^{2J}\Delta^{2w}_{2J}$$

$\leftrightarrow$

TST deformed AdS

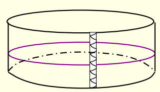


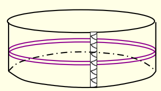
$\equiv$ AdS with twisted BC.

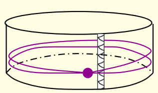
$E(\lambda)=J+$finite size corr.

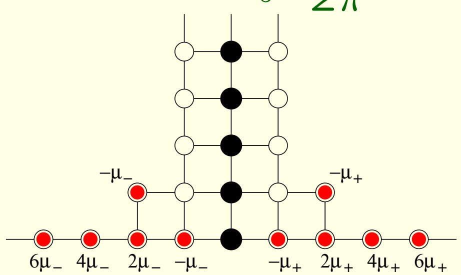
twisted groundstate

Twisted AdS: spectrum $Q=1,\ldots$, Tr $\to$ STr


$$=-\sum_Q \mathrm{STr}_Q(e^{i\gamma J})\int\frac{dp}{2\pi}e^{-e_Q(p)L}+\ldots$$


$$=\frac{1}{2}\sum_Q \mathrm{STr}_Q(e^{i\gamma J})^2\int\frac{dp}{2\pi}e^{-2e_Q(p)L}$$

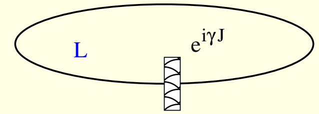

$$=\sum_{Q_1Q_2}\int\frac{dp_1}{2\pi}e^{-e_{Q_1}(p_1)L}\int\frac{dp_2}{2\pi}e^{-e_{Q_2}(p_2)L}i\partial_{p_1}\mathrm{STr}_2(e^{i\gamma J}\log S_{Q_1Q_2}(p_1,p_2))$$

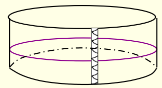
twisted TBA 

untwisted Y-system [Gromov .. '05]

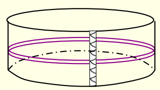
Application to twisted AdS/CFT with $L = J = 3$

Ground-state energy $E_0(L)$ with twisted BC ($e^{i\gamma J}$) in AdS CFT

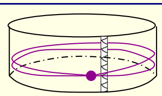


LO Lüscher corr.:  $= -\sum_Q \mathrm{STr}_Q(e^{i\gamma J}) \int \frac{dp}{2\pi} e^{-e_Q(p)L} + \ldots$ $\quad \gamma_\pm = \frac{J}{2}(\gamma_3 \pm \gamma_2)$

$$E_0^{(1)}(3) = -\sin^2\frac{\gamma_-}{2}\sin^2\frac{\gamma_+}{2}\left(3\zeta_3\frac{\lambda^3}{(4\pi)^3} - 15\zeta_5\frac{\lambda^4}{(4\pi)^4} + \frac{945}{16}\zeta_7\frac{\lambda^5}{(4\pi)^5} - \frac{3465}{16}\zeta_9\frac{\lambda^6}{(4\pi)^6} + \ldots\right)$$

NLO Lüscher corr.:  $= \frac{1}{2}\sum_Q \mathrm{STr}_Q(e^{i\gamma J})^2 \int \frac{dp}{2\pi} e^{-2e_Q(p)L}$

$$E_0^{(2,1)}(3) = \sin^2\frac{\gamma_-}{2}\sin^2\frac{\gamma_+}{2}\left(\frac{63}{4}\zeta_7\frac{\lambda^6}{(4\pi)^6}\right)$$

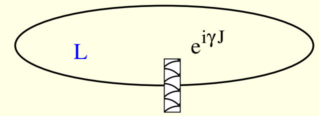
 $= \sum_{Q_1 Q_2}\int \frac{dp_1}{2\pi} e^{-e_{Q_1}(p_1)L} \int \frac{dp_2}{2\pi} e^{-e_{Q_2}(p_2)L} i\partial_{p_1}\mathrm{STr}(e^{i\gamma J}\log S_{Q_1 Q_2}(p_1, p_2))\ldots$

$$E_0^{(2,2)}(3) = \sin^2\frac{\gamma_-}{2}\sin^2\frac{\gamma_+}{2}\left(\sin^2\frac{\gamma_-}{2} + \sin^2\frac{\gamma_+}{2}\right)\left(\frac{15}{4}\zeta_3\zeta_5\frac{\lambda^6}{(4\pi)^6}\right)$$

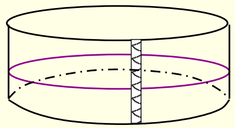
$$. \qquad -\sin^4\frac{\gamma_-}{2}\sin^4\frac{\gamma_+}{2}\left(9\zeta_3^2 + \frac{63}{16}\zeta_7\right)\frac{\lambda^6}{(4\pi)^6}$$

Conclusion

Ground-state energy $E_0(L)$ with twisted BC $(e^{i\gamma J})$



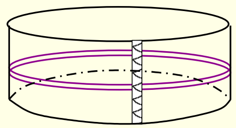
LO Lüscher correction:



$$= -\mathrm{Tr}_1(e^{i\gamma J}) \int \frac{dp}{2\pi} e^{-e(p)L} + \dots$$

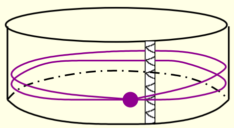
spectrum

NLO Lüscher correction:

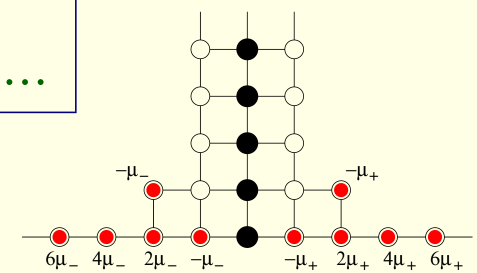


$$= \frac{1}{2}\mathrm{Tr}_1(e^{i\gamma J})^2 \int \frac{dp}{2\pi} e^{-2e(p)L}$$

scattering matrix



$$= \int \frac{dp_1}{2\pi} e^{-e(p_1)L} \int \frac{dp_2}{2\pi} e^{-e(p_2)L} i\partial_{p_1} \mathrm{Tr}_2(e^{i\gamma J} \log S(p_1, p_2))\dots$$

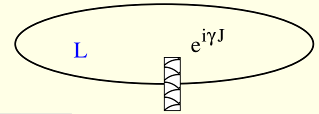


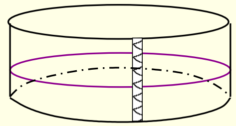
Twisted TBA equation: $\epsilon \to \epsilon + \mu$

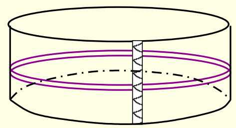
Double wrapping in twisted AdS/CFT for $L = 3$

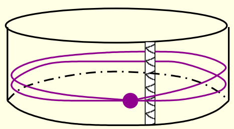
Conclusion

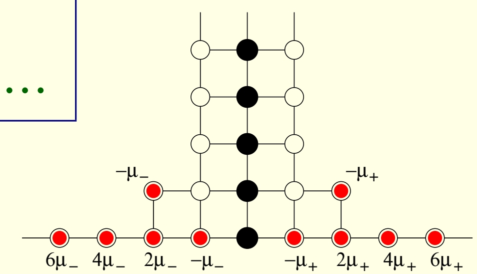
Ground-state energy $E_0(L)$ with twisted BC ($e^{i\gamma J}$)



LO Lüscher correction:  $= -\mathrm{Tr}_1(e^{i\gamma J}) \int \frac{dp}{2\pi} e^{-e(p)L} + \ldots$ spectrum

NLO Lüscher correction:  $= \frac{1}{2}\mathrm{Tr}_1(e^{i\gamma J})^2 \int \frac{dp}{2\pi} e^{-2e(p)L}$ scattering matrix

 $= \int \frac{dp_1}{2\pi} e^{-e(p_1)L} \int \frac{dp_2}{2\pi} e^{-e(p_2)L} i\partial_{p_1} \mathrm{Tr}_2(e^{i\gamma J} \log S(p_1,p_2))\ldots$



Twisted TBA equation: $\epsilon \to \epsilon + \mu$

Double wrapping in twisted AdS/CFT for $L = 3$

Outlook

Test double wrapping in gauge theory perturbation theory!

Test NLIEs by comparing to Lüscher corrections: O(N) [Balog.. '01] SS [Hegedus '04], AdS/CFT ?