Holographic Description of Hadrons from String Theory (Review)

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based on works with

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1 Introduction

Claim :

String theory can be a theory of hadrons

The string scale here is around 1 GeV.

I'm NOT saying QCD is wrong.

QCD

dual !

String theory
(in a certain curved background)

4 dim

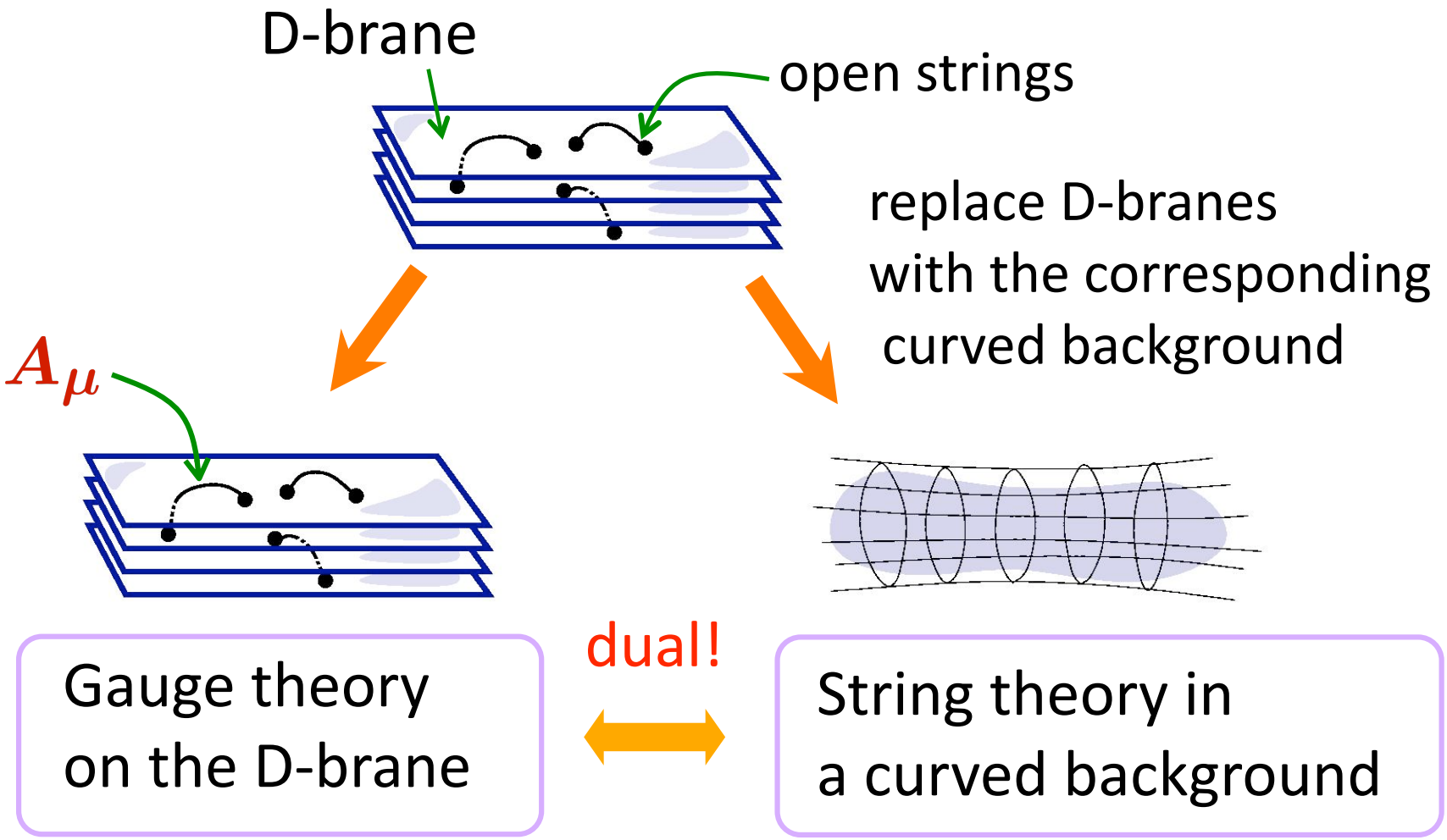
10 dim

" Holographic QCD "

"Gauge/String duality"

Gauge/String duality

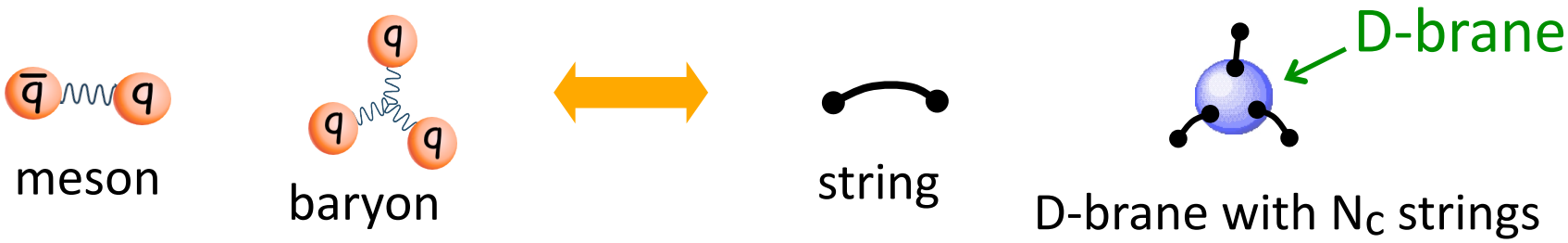
[Maldacena 1997, ...]



※ SUSY and conformal sym are not essential in this idea.

Claim :

QCD ⟷ (dual !) String theory (in a certain curved background)



- New description of hadrons
- Useful framework to understand / calculate various properties of hadrons
- Test of gauge/string duality using experimental data

Plan

✓ 1 Introduction

Part 1: Brief Overview

2 Brief review of the model

3 Results

Part 2: Mesons as Open strings

4 Excited mesons

5 Meson spectrum

6 Comparison with data

7 Conclusion

Part 1:

Brief Overview

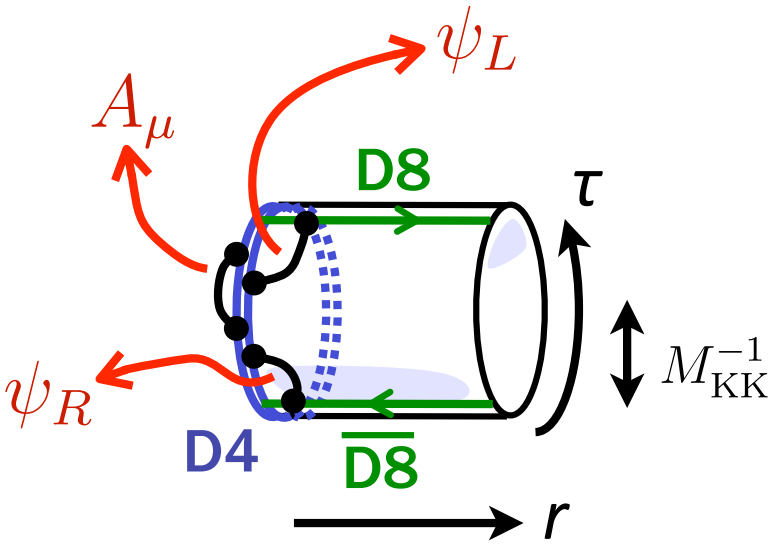
2 Brief review of the model

Brane configuration

[T. Sakai and S.S. 04]

	x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4 x N_C	O	O	O	O	O					
D8-$\overline{\text{D8}}$ x N_f	O	O	O	O		O	O	O	O	O

S^1 with ~~SUSY~~ boundary condition

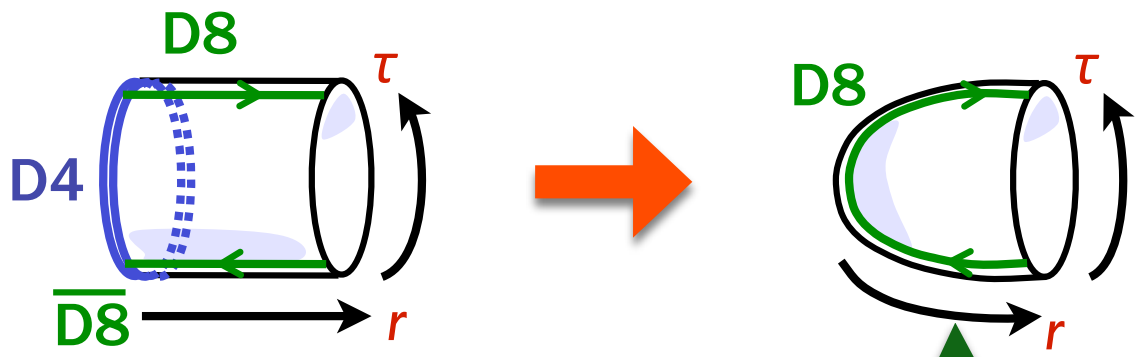


(radial direction of $x^{5\sim9}$)

4 dim $U(N_C)$ QCD
with N_f massless quarks
(at low energy)

Holographic description

- replace D4 with the corresponding curved background [Witten 98]
- D8 are treated as probe brane (assuming $N_c \gg N_f$)

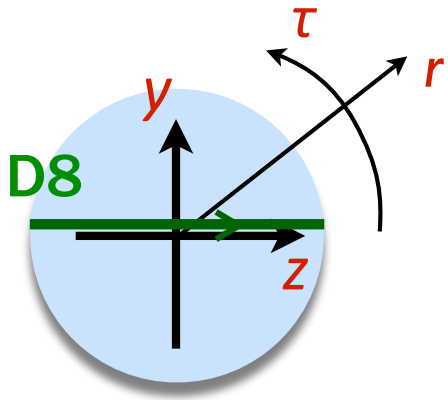


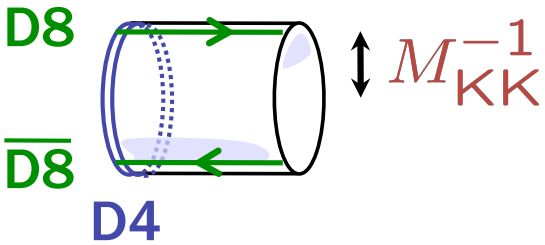
$$\mathbf{R}^{1,3} \times S^1 \times \mathbf{R}^5 \qquad\qquad \mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4 \quad \text{(topology)}$$

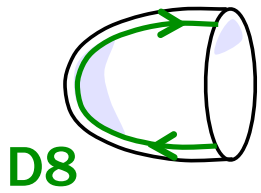
$x^{0\sim3}$ $\quad \tau \quad (r, \Theta^{1\sim4})$ $\qquad\qquad$ $x^{0\sim3}$ $\quad (r,\tau) \quad \Theta^{1\sim4}$

S^4

$$\begin{cases} y = r\sin\tau \\ z = r\cos\tau \end{cases}$$







(low energy)

$U(N_c)$ **QCD**
with N_f **massless quarks**

dual

Open + closed string theory
in this background

parameters :

$$\begin{cases} M_{\text{KK}} \sim \text{cut off scale} \\ \lambda = g_{\text{YM}}^2 N_c \end{cases}$$

('t Hooft coupling)

$$\begin{cases} \text{string coupling} & \propto 1/N_c \\ \text{string length} & \propto \lambda^{-1/2} \end{cases}$$

good description for

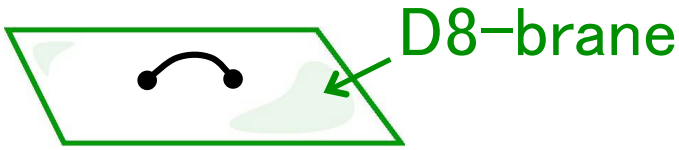
$$\begin{cases} \text{large} \quad N_c \\ \text{large} \quad \lambda \end{cases} \longleftrightarrow \text{low energy}$$

Do we have hadrons?

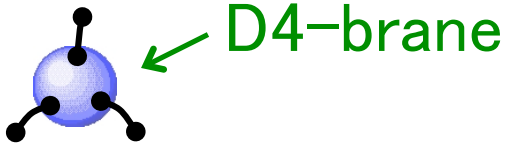
Particles in this system

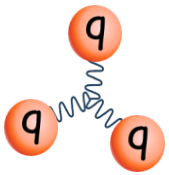
- Closed strings ↔ glueballs

- Open strings on D8 ↔ mesons



- D4 wrapped on S^4 ↔ baryons





N_c strings are attached

Surprise :

A lot of properties of hadrons can be extracted using this new description!

Caution :

- We have only estimated the leading terms in the $1/N_c$ and $1/\lambda$ expansions.
- M_{KK} is around 1 GeV.
- Quark masses are neglected.

We should not expect too much.
But, don't be too pessimistic!

- Meson effective theory is written as a 5 dim $U(N_f)$ YM-CS theory in a curved background.

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$k(z) = 1 + z^2$

CS5-form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr}\left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2\right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$\mu, \nu = 0 \sim 3$

$h(z) = (1 + z^2)^{-1/3}$

[See also, Son-Stephanov 2003]

$$A_\mu(x^\mu, z) = \sum_{n \geq 1} B_\mu^{(n)}(x^\mu) \psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n \geq 0} \varphi^{(n)}(x^\mu) \phi_n(z)$$

complete sets of functions of z

$\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

$$S_{\text{5dim}}(A) = S_{\text{4dim}}(\pi, \rho, a_1, \rho', a_1', \cdots)$$

traditional meson effective action

- A lot of old models are reproduced without making any phenomenological assumptions!

Skyrme model [Skyrme 1961]

Vector meson dominance [Sakurai 1960, Gell-Mann-Zachriasen 1961]

Gell-Mann Sharp Wagner model [Gell-Mann –Sharpe-Wagner 1962]

Hidden local symmetry [Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]

next slide

- masses and couplings roughly agree with experimental data.

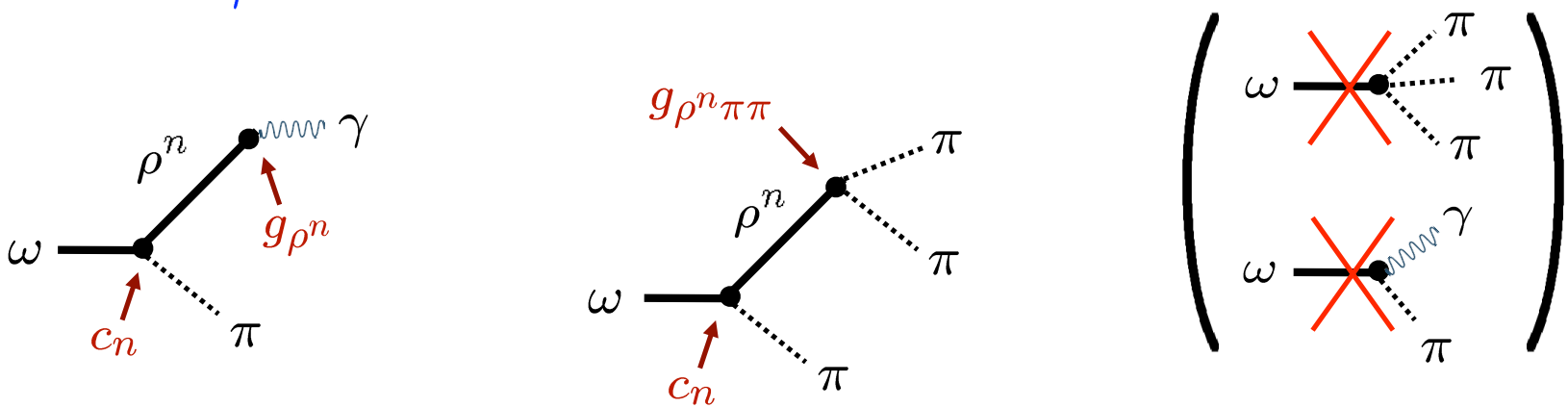
mass	ρ	a_1	ρ'
exp.(MeV)	776	1230	1465
our model	[776]	1189	1607
ratio	[1]	1.03	0.911

input

coupling	our model	experiment
f_π	[92.4 MeV]	92.4 MeV
L_1	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	4.81	5.99
g_ρ	0.164 GeV^2	0.121 GeV^2
$g_{a_1\rho\pi}$	4.63 GeV	$2.8 \sim 4.2$ GeV

ω meson decay ($\omega \to \pi^0\gamma$ and $\omega \to \pi^0\pi^+\pi^-$)

- Our model predicts that the relevant diagrams for $\omega \to \pi^0\gamma$ and $\omega \to \pi^0\pi^+\pi^-$ are



➡ Exactly the same as the GSW model !

[Gell-Mann -Sharp-Wagner 1962]

- Furthermore, we find

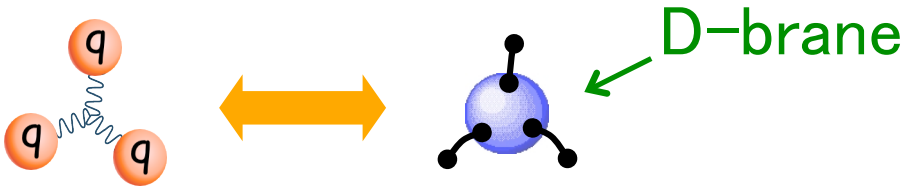
$$\Gamma(\omega \to \pi^0\gamma) = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2}\left(\sum_{n=1}^{\infty}\frac{c_n\, g_{\rho^n}}{m_{\rho^n}^2}\right)^2 |\mathbf{p}_\pi|^3 = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2}\, {g_{\rho\pi\pi}}^2 |\mathbf{p}_\pi|^3$$

➡ reproduces the proposal given by Fujiwara et al !

[Fujiwara-Kugo-Terao-Uehara-Yamawaki 1985]

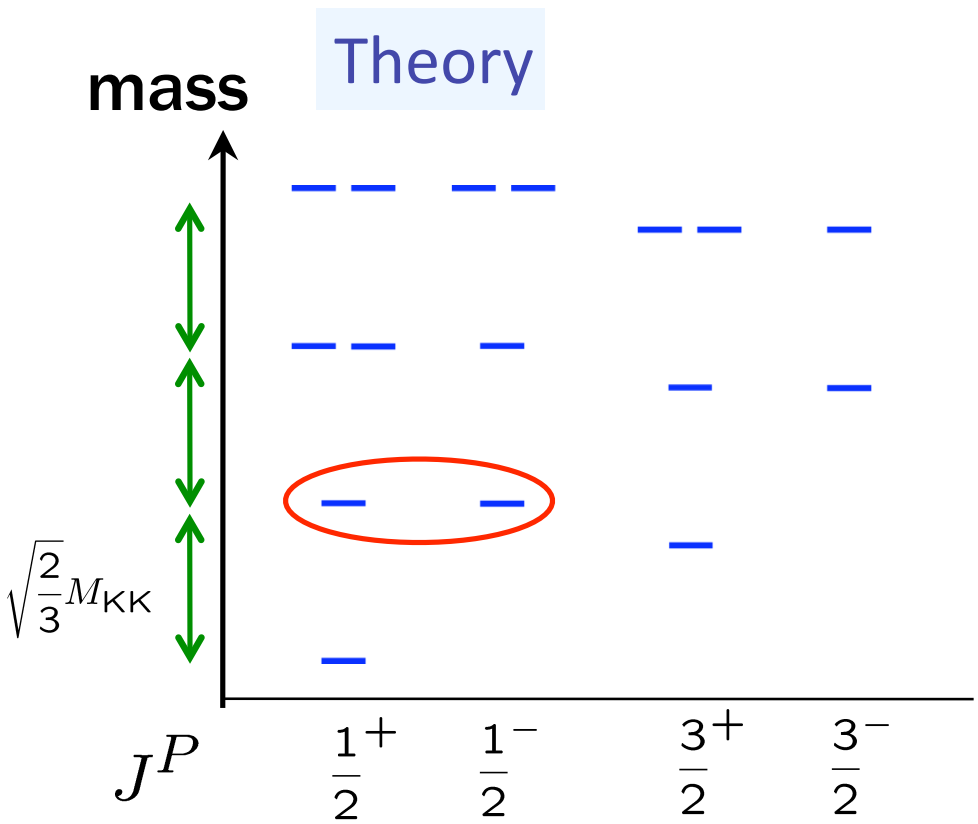
Baryon spectrum

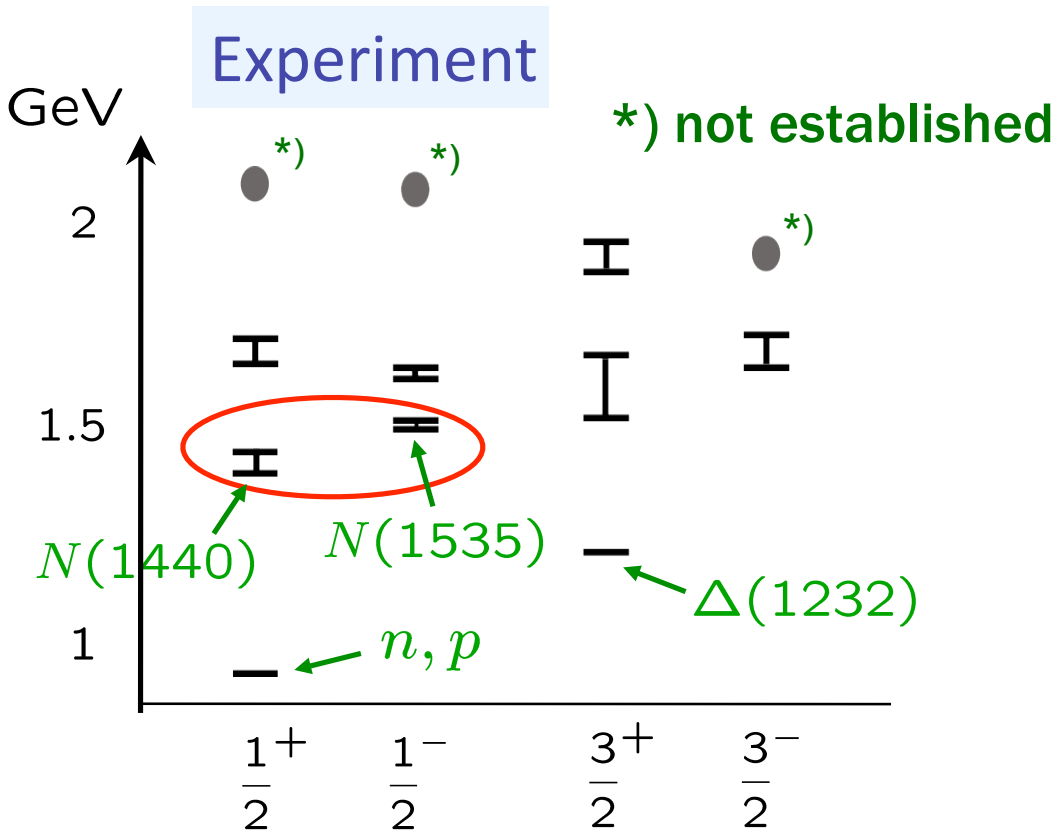
[Hata-Yamato-Sakai-S.S. 2007]



= soliton in the 5dim theory

We can analyze the spectrum, magnetic moments, charge radii by quantizing this soliton cf) Skyrme model





※ Quantitatively, not very successful.
If we use M_ρ to fix M_{KK}, we obtain $\sqrt{2/3}\,M_{\rm KK} \simeq 775\ {\rm MeV}$

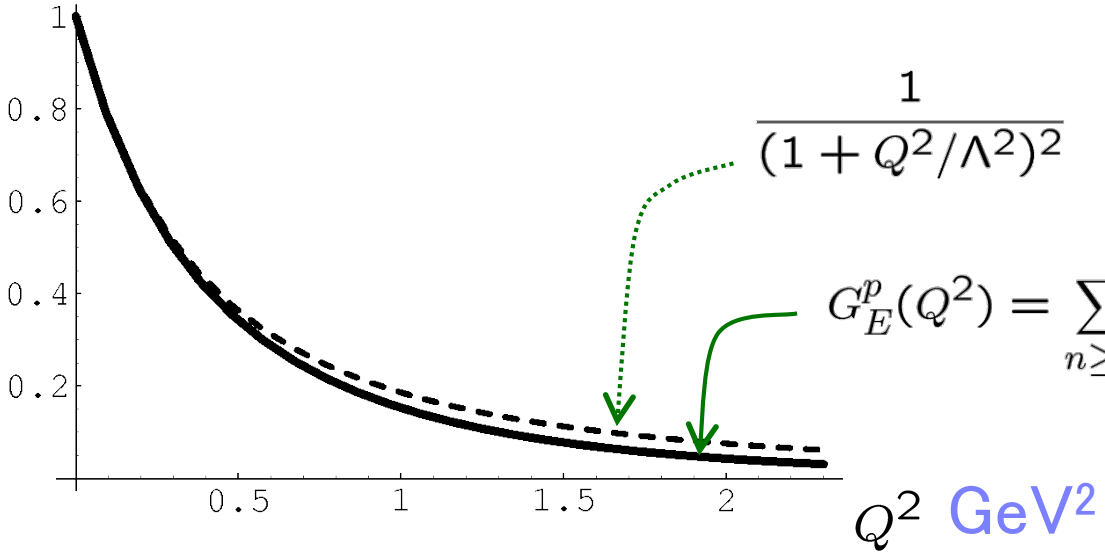
Properties of nucleons

[Hashimoto-Sakai-S.S. 2008]

[See also,
Hong-Rho-Yee-Yi 2007,
Hata-Murata-Yamato 2008,
Kim-Zahed 2008,
Pomarol-Wulzer 2008, ...]

	our result	exp.
$\langle r^2 \rangle_{I=0}^{1/2}$	0.74 fm	0.81 fm
$\langle r^2 \rangle_{I=1}^{1/2}$	0.74 fm	0.94 fm
$\langle r^2 \rangle_{A}^{1/2}$	0.54 fm	0.67 fm
$g_{I=0}$	1.7	1.8
$g_{I=1}$	7.0	9.4
g_A	0.73	1.3

nucleon ele-mag form factor



$$\frac{1}{(1+Q^2/\Lambda^2)^2}$$ dipole ($\simeq$ experiment)

$$G_E^p(Q^2) = \sum_{n\geq 1} \frac{g_{v^n} g_{v^n NN}}{Q^2 + m_n^2}$$ our result

Part 2:

Mesons as Open Strings

based on **arXiv:1005.0655** Imoto-Sakai-S.S.

4 Excited mesons

- **mesons** (N_f = 2, Isovector)

parity

spin

J^{PC}

charge conjugation

mass (MeV)

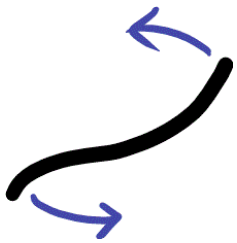
△ ... not established

considered in Part 1

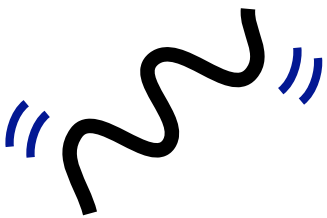
$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	985	1474				
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				
$1^{+-}(b_1)$	1230					
$1^{-+}(\pi_1)$	1376	1653				
$2^{++}(a_2)$	1318	$1732^{\triangle}$				
$2^{-+}(\pi_2)$	1672	1895	$2090^{\triangle}$			
$3^{--}(\rho_3)$	1689	$1990^{\triangle}$	$2250^{\triangle}$			
$4^{++}(a_4)$	2001					
$5^{--}(\rho_5)$	$2330^{\triangle}$					
$6^{++}(a_6)$	$2450^{\triangle}$					

Q: What about the other mesons?

- Intuitively,



rotating open string → higher spin meson



vibrating open string → heavier meson

Let's consider excited open string states.

5 Meson spectrum

Consider open strings attached on D8

't Hooft coupling

$\lambda \sim (\text{curvature radius}/l_s)^2$

Strategy

1. Consider flat space-time, (justified when $\lambda \gg 1$)
   and quantize the open strings attached on D8.

   space-time: $\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$ D8-brane: $(x^\mu, z) \times S^4$
   (topology) $x^{0\sim3}$ (z,y) $x^{6\sim9}$

   In the flat space-time limit,
   $S^4 \Rightarrow R^4$, $SO(5) \Rightarrow$ rotation and translation of $x^{6\sim9}$

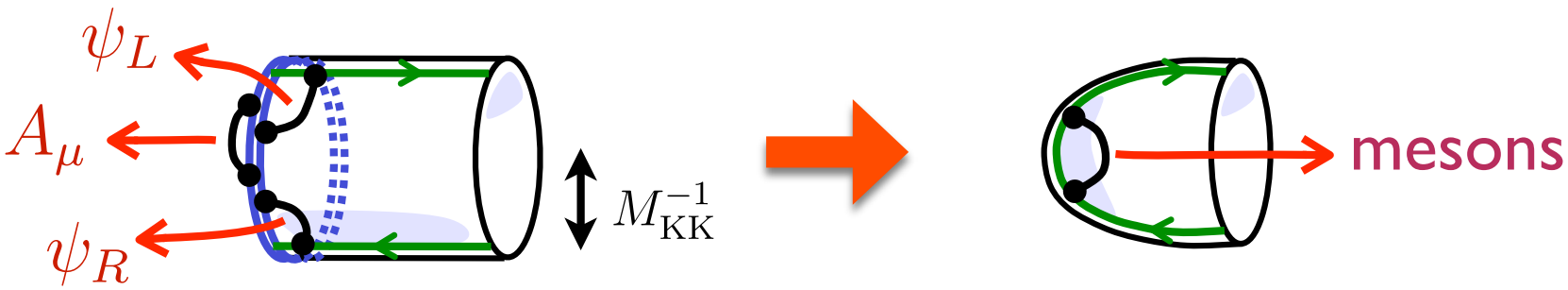
2. Pick up the $SO(5) \rtimes \mathbf{Z}_2$ invariant states. ← See next slide

   → reduced to 5 dim: (x^μ, z)

3. Recover the z dependence of the background (perturbatively).

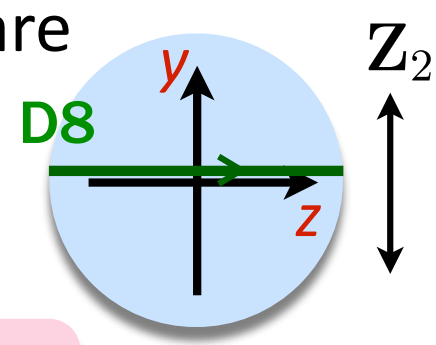
QCD mesons vs artifacts

- Our brane config. is invariant under $SO(5) \curvearrowright S^4$
- quarks and gluons are invariant under $SO(5)$
  (non-invariant states are massive KK modes)



- Bound states of quarks and gluons are $SO(5)$ invariant
  (non-invariant states are artifacts made by unwanted massive modes)
- Similarly, we can show that quarks and gluons are invariant under $\mathbf{Z}_2$ sym generated by $I_{y9}(-1)^{F_L}$

  I_{y9}: $(y, x^9) \to (-y, -x^9)$ $\quad (\tau \to -\tau)$



$\Rightarrow$ Consider $SO(5) \rtimes \mathbf{Z}_2$ invariant states

First excited massive modes

$SO(5) \rtimes \mathrm{Z}_2$ invariant states (light cone gauge):

$(I = 2, 3, z)$

- $\psi^I_{-3/2}|0\rangle_{\rm NS}$
- $\alpha^{(I}_{-1}\psi^{J)}_{-1/2}|0\rangle_{\rm NS}$
- $\alpha^{[I}_{-1}\psi^{J]}_{-1/2}|0\rangle_{\rm NS}$
- $\psi^I_{-1/2}\psi^J_{-1/2}\psi^K_{-1/2}|0\rangle_{\rm NS}$
- $\alpha^y_{-1}\psi^y_{-1/2}|0\rangle_{\rm NS}$
- $\sum_{a=\underbrace{6,7,8,9}_{S^4}} \alpha^a_{-1}\psi^a_{-1/2}|0\rangle_{\rm NS}$

5dim field	$SO(4)$ little gr
h_{MN} $(M, N = 1, 2, 3, z)$	$\square\square$
A_{MNP}	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$
$\varphi^{[1]}$	1
$\varphi^{[2]}$	1

KK decomposition along z direction

$$h_{MN}(x^\mu, z) = \sum_{n=0}^{\infty} h_{MN}^{(n)}(x^\mu)\phi_n(z) \qquad \text{etc.}$$

lowest modes: (i, j, k = 1, 2, 3)

	$h_{ij}^{(0)}$	$h_{iz}^{(0)}$	$h_{zz}^{(0)}$	$A_{ijk}^{(0)}$	$A_{ijz}^{(0)}$	$\varphi^{[1,2](0)}$
J^{PC}	2^{++}	1^{+-}	0^{++}	0^{-+}	1^{--}	0^{++} x 2

Second excited massive mode

lowest modes:

J^{PC}	3^{--}	2^{++}	2^{--}	2^{-+} x 2	1^{--} x 7	1^{++} x 3	1^{+-} x 4	1^{-+}	0^{++} x 2	0^{-+} x 6

Mass formula (naive shortcut)

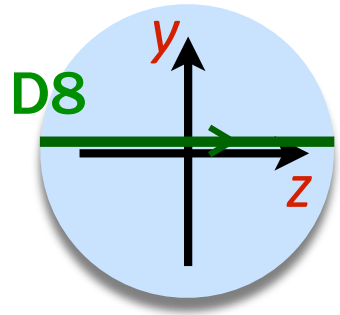
- Flat space-time limit:

$$m_0^2 = \frac{N}{\alpha'} \qquad \left(\alpha'^{-1} = \frac{4}{27}\lambda M_{\rm KK}^2\right)$$

$N = 0, 1, 2, \cdots$: excitation level

- Massive particle in curved space-time

$$S = -m_0 \int \sqrt{g_{tt}}\, dt = -m_0 (1+z^2)^{1/4} \int dt$$



→ particle in potential: $V(z) = m_0(1+z^2)^{1/4}$

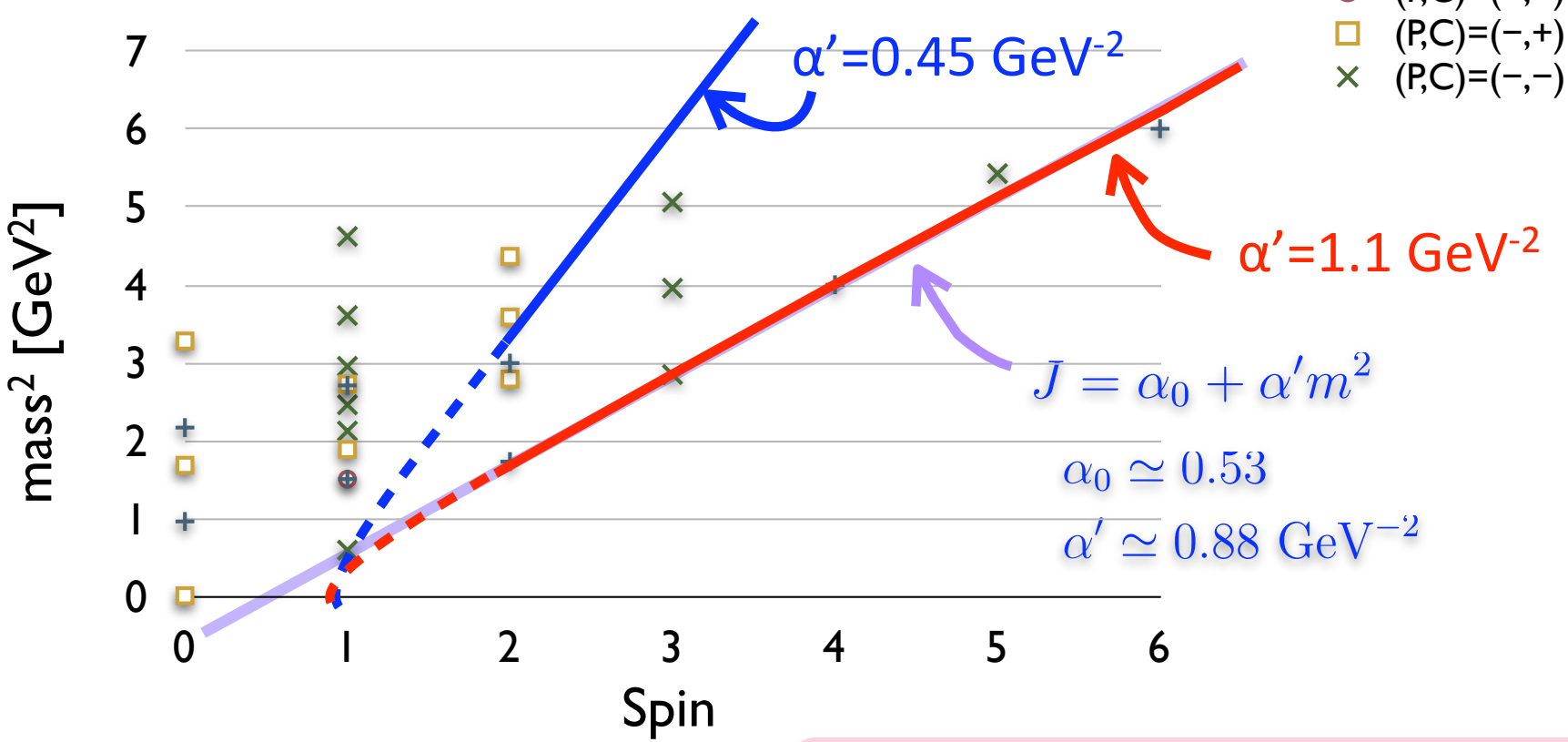
→ $$M_n \simeq m_0 + \frac{1}{\sqrt{2}}\left(n + \frac{1}{2}\right) M_{\rm KK} + \mathcal{O}(\lambda^{-1/2})$$

$n = 0, 1, 2, \cdots$

harmonic oscillator approx.

- More careful analysis shows that the $O(1)$ term is not affected by the RR-flux, α' correction, etc.
(This argument only works for open string states.)

Regge trajectory



$$M_n \simeq \sqrt{\frac{N}{\alpha'}} + \frac{1}{\sqrt{2}}\left(n + \frac{1}{2}\right) M_{\mathrm{KK}}$$

(for $N \geq 1$)

$$J \simeq 1 + \alpha' M^2 - \frac{\alpha'}{\sqrt{2}} M_{\mathrm{KK}} M + \mathcal{O}(\lambda^{-1})$$

$N = J - 1,\ n = 0$

- If we use f_π to fix α', we obtain α' = 0.45 GeV^{-2}. This is unfortunately too small.
- If we set α'=1.1 GeV^{-2} we get very good fit.

6 Comparison with data

Massless mode

[T.Sakai and S.S. 04]

	$B_\mu^{(1)}$	$B_\mu^{(2)}$	$B_\mu^{(3)}$	$B_\mu^{(3)}$	...	$\varphi^{(0)}$
J^{PC}	1^{--}	1^{++}	1^{--}	1^{++}	...	0^{-+}
	ρ	a_1	ρ'	a'_1	...	π
mass (MeV)	[776]	1189	1607	2024	...	0

used to fix M_{KK}=949 MeV

?

experiment:

$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				

First excited states (N=1, n=0)

J^{PC}	2^{++}	1^{+-}	1^{--}	0^{-+}	0^{++} x 3

2^{++}, 1^{+-}, 0^{-+}, 0^{++} cannot be N=0
⇒ good candidates for N=1

?

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	*~~985~~	1474				
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				
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$6^{++}(a_6)$	$2450^{\triangle}$					

: N=0
: N=1

- degenerate around 1300 MeV
- *a_0(980) is considered to be a four quark state.
- interpretation of the degeneracy is not clear

Second excited states (N=2, n=0)

J^{PC}	3^{--}	2^{++}	2^{--}	2^{-+} x 2	1^{--} x 7	1^{++} x 3	1^{+-} x 4	1^{-+}	0^{++} x 2	0^{-+} x 6

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	~~985~~	1474				
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				
$1^{+-}(b_1)$	1230					
$1^{-+}(\pi_1)$	~~1376~~ *	1653				
$2^{++}(a_2)$	1318	$1732^{\triangle}$				
$2^{-+}(\pi_2)$	1672	1895	$2090^{\triangle}$			
$3^{--}(\rho_3)$	1689	$1990^{\triangle}$	$2250^{\triangle}$			
$4^{++}(a_4)$	2001					
$5^{--}(\rho_5)$	$2330^{\triangle}$					
$6^{++}(a_6)$	$2450^{\triangle}$					

: N=0
: N=1
: N=2

- degenerate around 1700 MeV
- ★: prediction ?
- * π_1 (1400) is claimed to be a four quark state. (could be hybrid)

Summary

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	~~985~~	1474	★			
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				
$1^{+-}(b_1)$	1230	★				
$1^{-+}(\pi_1)$	~~1376~~	1653				
$2^{--}(\rho_2)$	★					
$2^{++}(a_2)$	1318	$1732^{\triangle}$				
$2^{-+}(\pi_2)$	1672	1895	$2090^{\triangle}$			
$3^{--}(\rho_3)$	1689	$1990^{\triangle}$	$2250^{\triangle}$			
$4^{++}(a_4)$	2001					
$5^{--}(\rho_5)$	$2330^{\triangle}$					
$6^{++}(a_6)$	$2450^{\triangle}$					

: N=0

: N=1

: N=2

: 4 quarks

I think this is non-trivial.
What do you think?

7 Conclusion

- String theory can be a theory of hadrons.
- A lot of properties of hadrons can be extracted using this new description!
- The predictions seem to agree reasonably well with experimental data.
  Much better than expected!