Nonlinear W_∞ Algebra as Asymptotic Symmetry of AdS_3 Higher Spin Gauge Theory

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joint works with

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Higher Spin Gravity Theory

* $s \geq 2$

* [Aragone + Deser; 1979]
minimal coupling is inconsistent
in flat space for $s > 3/2$

why?

δ (higher spin) $\simeq$ (Riem) * $(\cdots)$ + $\cdots$

$\delta(\sqrt{-g}\,R) \simeq (\nabla \cdot \text{Ricci}) + \cdots$

under higher spin gauge transf.

can NOT cancel each other

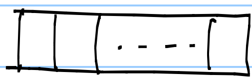
* ways out

- [Vasiliev] put on (A)dS space

- [Aragone + Deser] d = (2+1)

(Weyl)$_{d=3}$ = 0 identically

Notes on HS theory

* field $\varphi_{(\mu_1\cdots\mu_s)}$ 

* parameter $\xi_{(\mu_1\cdots\mu_{s-1})}$

* free eqn of motion: $(\Lambda=0)$

$$\Box\varphi_{(\mu_1\cdots\mu_s)} - \partial_{(\mu_1}\partial^{\lambda}\varphi_{\lambda\mu_2\cdots\mu_s)}$$

$$+\,\partial_{(\mu_1}\partial_{\mu_2}\varphi_{\mu_3\cdots\mu_s)\lambda}{}^{\lambda} = 0$$

* gauge orbits

$$\delta\varphi_{(\mu_1\cdots\mu_s)} = \partial_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}$$

$$\xi_{\mu_1\cdots\mu_{s-3}\lambda}{}^{\lambda} = 0$$

* [Fronsdal] if $\varphi_{\mu_1\cdots\mu_{s-4}\nu\lambda}{}^{\nu\lambda} = 0$,

$$S = \frac{1}{2}\int \varphi^{(\mu_1\cdots\mu_s)}\Big[(EOM)_{\mu_1\cdots\mu_s}$$

$$-\,\frac{1}{2}\eta_{\mu_1\mu_2}(EOM)_{\mu_3\cdots\mu_s\lambda}{}^{\lambda}\Big]$$

* modification with $\Lambda < 0$

- covariantize $\partial_\mu \to \nabla_\mu (\Gamma_{AdS})$
- nonzero curvature

$$[\nabla_\mu, \nabla_\nu] V_\rho = R_{\mu\nu\rho\sigma} V^\sigma \sim \mathcal{O}\left(\frac{1}{L^2}\right)$$

$$\therefore \ (\text{EoM})\big|_{\Lambda<0}$$

$$= (\text{EoM})\big|_{\Lambda=0} - \frac{1}{L^2}(\text{Mass})$$

where

$$\boxed{\begin{aligned}&(\text{Mass})\\&= [s^2 + (D-6)s - 2(D-3)]\,\varphi_{(\mu_1\cdots\mu_s)}\\&+ 2\, g_{(\mu_1\mu_2}\varphi_{\mu_3\cdots\mu_s)\lambda}{}^{\lambda}\end{aligned}}$$

- action is the same

Frame description of HS fields

* $\varphi_{(\mu_1 \cdots \mu_s)}$

$$\longrightarrow \quad e_\mu^{a_1 \cdots a_{s-1}} \oplus \omega_\mu^{a_1 \cdots a_{s-1}}$$

Supplemented by

$$\delta e_\mu^{a_1 \cdots a_{s-1}} = D_\mu \xi^{a_1 \cdots a_{s-1}} + \bar{e}_\mu^{\,b}\, \omega_b^{\;a_1 \cdots a_{s-1}}$$

↑ background spacetime

<u>then</u>

$$\boxed{\varphi_{(\mu_1 \cdots \mu_s)} \equiv \frac{1}{s}\left(\bar{e}_{\mu_1}^{\,a_1} \cdots \bar{e}_{\mu_{s-1}}^{\,a_{s-1}}\right) \cdot e_{\mu_s a_1 \cdots a_{s-1}}}$$

Stringy Motivation

* AdS/CFT [Witten '01] [Sundborg '94]

$$g_{st},\ (R/l_s) \Longleftrightarrow g_{YM}^2 N,\ \frac{1}{N^2}$$

$$\boxed{\left(\frac{R}{l_s}\right) = \left(g_{YM}^2 N\right)^{1/4}}$$

AdS_5/CFT_4

⇝ weak 't Hooft coupling means

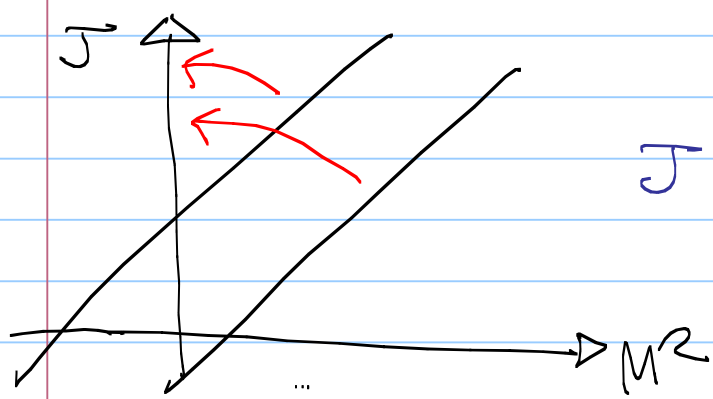
$l_s^2/R^2 \gg 1$

equiv

$T_{string} \ll \frac{1}{R^2}$

∴ tensionless string in fiducial AdS-space

⇝ (infinite) tower of Regge trajectories fall down



$$J = l_s^2 M^2 + J_0$$

* consequences to microstates

- asymptotic symmetries match between AdS and CFT of finite dimensionality

(e.g.) AdS_3 / CFT_2

$Vir \oplus \overline{Vir}$ as asymptotic symm.

with $\boxed{c_{CFT} = 3L/2G_N}$

↪ what happens to the asymptotic symmetry as 't Hooft coupling is lowered?

Answering this requires

quantization of gravity with effective degrees of freedom

$\oplus$

fate of black holes

In this talk

higher-spin gauge theory

$\subset$ tensionless string theory

$\Updownarrow$

critical string on AdS

Results

For (2+1) d higher spin AdS gravity with $G = hs(1,2:\mathbb{R})$

(i) G_∞ = nonlinear W_∞

(ii) $c = 3L / 2G_N$ again

$HS(1,2;\mathbb{R})$

- ξ^1, ξ^2 : commuting twistors of $Sp(2,\mathbb{R})$

- $\mathcal{P}(\xi) = \{$ polynomials of even degrees in twistors $\}$

- $\mathrm{Tr}\, f(\xi) \equiv 2 f(\xi)\big|_{\xi \to 0} = 2f(0)$

  for $f \in \mathcal{P}$

- $(f * g)(\xi)$

$$\equiv \exp\left[i\, \epsilon^{ab} \partial_{\xi^a} \partial_{\xi'^b} \right] f(\xi)\, g(\xi')\Big|_{\xi = \xi'}$$

- $[f, g]_*$

$$\equiv \frac{1}{2i} \left[f * g - g * f \right]$$

$$= \sin\left(\partial_\xi \wedge \partial_{\xi'} - \partial_{\xi'} \wedge \partial_\xi \right) f(\xi)\, g(\xi')\Big|_{\xi = \xi'}$$

- $(f, g) \equiv \mathrm{Tr}\,(f * g)$

then

$$\boxed{hs(1,2;\mathbb{R}) = \mathcal{P} / \{1\}.}$$

$\mathfrak{sl}(2,\mathbb{R})$ subalgebra

- $\mathcal{P}_2 = \{\text{degree-2 polynomials}\}$

$X_{11} = \frac{1}{2}(\xi_1)^2$ - e lowest weight

$X_{12} = \xi_1 \xi_2$ - h

$X_{22} = \frac{1}{2}(\xi_2)^2$ f highest weight

- $[X_{11}, X_{12}] = 2X_{11}$

$[X_{11}, X_{22}] = X_{12}$

$[X_{12}, X_{22}] = 2X_{22}$

- $(X_{12}, X_{12}) = 2$

$(X_{11}, X_{22}) = -1$

$(X_{22}, X_{11}) = -1$

$\#(\xi_1) = \#(\xi_2)$ for nonzero bilinear form

$hs(1,2;\mathbb{R})$ representations

- $hs(1,2;\mathbb{R}) = \bigoplus_{k\geq 1} D_k$

  where

  D_k = spin-k representation via P_{2k}

- basis

  $X_{(p,q)} \equiv \frac{1}{p!_0} \frac{1}{q!_0} (\xi_1)^p (\xi_2)^q$ $(p+q=2k)$

- D_k yields asymptotically generators M_{k+1} of spin $s=(k+1)$

  (conformal spin $= k$)

(ex) $sl(3,\mathbb{R}) = \{X_{(2)}, X_{(4)}\}$

$(2\cdot 1+1)+(2\cdot 2+1) = 8$ generators

- 'accidental' closed subalgebra of $hs(1,2;\mathbb{R})$

3d Gravity as Chern-Simons

- [Achucarro + Townsend]

AdS (super) gravity by CS gauge theory formulation

$$G = SO(2,2) \cong SO(2,1) \times SO(2,1)$$

$$S_{grav}[e]$$

$$= S_{CS}(A) - S_{CS}(\bar{A})$$

$$S_{CS}(A) = \frac{k}{4\pi}\int_{M_3} \mathrm{Tr}\left(A dA + \frac{2}{3} A^3\right)$$

$$k = L/4G_N, \quad (c = 6k)$$

$$A \in SO(2,1), \quad \bar{A} \in SO(2,1)$$

$$\begin{cases} A = \omega + \frac{1}{L} e \\ \bar{A} = \omega - \frac{1}{L} e \end{cases}$$

- higher-spin generalization [Blencowe]

$$(A, \bar{A}) \Longrightarrow (\Gamma, \bar{\Gamma})$$

where

$$\Gamma = \sum_{s=2}^{\infty} A^{a_1 \cdots a_{s-1}} X_{(s-1)}$$

- check : $sl(2,\mathbb{R})$ truncation

$$A = \omega + \frac{1}{L} e \; ; \quad \bar{A} = \omega - \frac{1}{L} e$$

$$S_{CS} = \frac{1}{8\pi G_N} \int \left(\frac{1}{2} eR + \frac{1}{L^2} e + \cdots \right)$$

EOM:
$$de + \omega \wedge e \equiv T = 0$$
$$d\omega + \frac{1}{2} \omega \wedge \omega + \frac{1}{2L^2} e \wedge e = 0$$

- $sl(3,\mathbb{R})$ truncation

$$A_3^{ab} = \omega^{ab} + \frac{1}{L} e^{ab} ; \quad \bar{A}_3^{ab} = \omega^{ab} - \frac{1}{L} e^{ab}$$

$$\mathcal{L}_{HS_3} = \frac{1}{2} e^a \wedge [\omega^{bc} \wedge \omega^{de} + e^{bc} \wedge e^{de}] \epsilon_{abc} \eta_{de}$$
$$+ \frac{1}{2} e^{ab} \wedge [d\omega_{ab} + \omega_a \wedge \omega_b]$$

EOM:
$$\begin{cases} de^2 + 2(e_1 \wedge \omega^2 + \omega_1 \wedge e^2) = 0 \\ d\omega^2 + 2(\omega_1 \wedge \omega^2 + \frac{1}{L^2} e_1 \wedge e^2) = 0 \end{cases}$$

Asymptotic Symmetries

- By inspection & intuition, extend $s=2$ case of $G_{\infty} = V_{1r} \oplus \overline{V_{1r}}$

- For $s=2$, asymptotic AdS means $sl(2,\mathbb{R})$ connection behaves

$$A_{s=2} \simeq \left[-\frac{2\pi}{k} \cdot \frac{1}{r} L(\phi,t) X_{11} + 1 \cdot r X_{22} \right] dx^+ - \left[\frac{1}{2r} X_{12} \right] dr$$

$$x^{\pm} \equiv (t \pm \phi) \quad \text{at} \quad \partial AdS_3$$

- radial gauge

$$A_{s=2} \to \underset{s=2}{\Delta} \equiv \Omega \cdot (\partial + A_{s=2}) \cdot \Omega^{-1}$$

with $\Omega = \begin{pmatrix} \sqrt{r} & 0 \\ 0 & -\frac{1}{\sqrt{r}} \end{pmatrix}$

$\rightsquigarrow$

$$\Delta_{s=2} = \Delta_+ dx^+$$

only this part

$$= \left[1 \cdot X_{22} - \frac{2\pi}{k} \cdot L(\phi,t) \cdot X_{11} \right] dx^+$$

constant

no r-dep.

- $s > 2$ extension

by intuition, working out similar steps.

highest weight

$$\Delta = \Big[1 \cdot X_{22} - \frac{2\pi}{k}\Big(L(\phi,t) X_{11} + 12 \cdot M(\phi,t) X_{1111} + \cdots \Big) \Big] dx^+$$

lowest weights

- asymptotic symmetry G_∞ is the set of bulk gauge transform that preserve the structure of (radial gauge) connection 1-form.

$$\delta \Delta = \partial_+ \Lambda + [\Delta, \Lambda]$$

(i) $\Lambda = \Lambda(x^+, \cancel{x^-})$

to maintain $\Delta_- = 0$

(ii) $\Lambda = \varepsilon(x^+) X_{22} + \sum_{s \geq 2} \eta_{s+1}(x^+) X_{(0,2s)} + \lambda$

(iii) λ = fixed by asymptotic condition in terms of ε, η_{s+1}

Since x is eliminated as a function of ε, η_{s+1} and its fields are linearly dependent on ε, η_{s+1}, it turns out

gauge transform is field-dependent (through $\lambda(\varepsilon, \eta_{s+1})$)

→ gauge algebra becomes NON LINEAR

Details of Gauge Algebra

- general principle of gauge theory asserts that asymptotic symmetry is generated in equal-time Poisson bracket by

$$G[\Lambda] = \int_{M_3} \mathrm{Tr}(\Lambda \cdot \mathcal{G}) + S_\infty$$

where

(i) $\mathcal{G} = g_A$ = CS Gauss' law ;

(ii) S_{∞} = Regge–Teitelboim term to have $\delta G[\Lambda]$ no ∂'s

Recalling $\mathrm{Tr}[(HW)(LW)] \neq 0$

$$\boxed{G[\Lambda] = \oint_{\partial M_3} (\varepsilon \cdot L + \eta_3 M + \cdots) \quad + \; (0 \text{ by on-shell})}$$

$\checkmark$

- with

$$\Delta = X_{22} + \sum_{k \geq 1} N^{(2k,0)} X_{(2k,0)}$$

$$\Lambda = \sum_{k} \rho_{(0,2k)} X_{(0,2k)} \leftarrow (\varepsilon, \eta_{5+1} \text{ part})$$

$$+ \sum_{k,\, l \neq 0} \rho_{(l,2k-l)} X_{(l,2k-l)}$$

$\leftarrow$ (eventually dependent part)

require

$$\delta \Delta = \partial_{+} \Lambda + [\Delta, \Lambda]$$

Solve this recursively in (k, l)

and find

(i) λ's are determined by $(\varepsilon, \eta_{5+1})$

(ii) recursion gives another source of nonlinearity

Nonlinear $W(\infty)$

- From $\delta N_{(2k,0)} = \underbrace{G_{(2k,0)}(\varepsilon,\eta)}$ $\quad f_{(0,2k)} \parallel$

$$G[\Lambda] = \oint \sum_{k=1}^{\infty} f_{(0,2k)} N_{(2k,0)}$$

Thus,

$$\delta N_{(2k,0)} = \{ N_{(2k,0)}, G[\Lambda] \}_{PB}$$

$$= \underbrace{\oint \sum_{k'=1}^{\infty} f_{(0,2k')} \{ N_{(2k,0)}, N_{(2k',0)} \}_{PB}}$$

$$\therefore C_{(2k,0)} = \sum_{k'} f_{(0,2k')} C_{(2k',0)}$$

we can read off

$$\boxed{\{ N_{(2k,0)}, N_{(2l,0)} \}_{PB} = \left(\text{nonlinear } \cancel{\text{ftns}} \text{ of } N_{(2k',0)}\text{'s} \right)}$$

Claim:

This algebra is nonlinear $W(\infty)$

Why?

① $\mathcal{A}$ contains $Vir \oplus \overline{Vir}$ with

$$c_2 = \frac{k}{4\pi}$$

$$\boxed{\{L(\phi), L(\phi')\} = -\frac{k}{4\pi}\delta'''(\phi-\phi') + (L(\phi)+L(\phi'))\delta'(\phi-\phi')}$$

② $M_{s+1} \simeq N(2s,0)$ has conformal weight $(s+1)$ with

$$\boxed{\{L(\phi)\, M_{s+1}(\phi')\} = (M_{s+1}(\phi) + s\, M_{s+1}(\phi'))\delta'(\phi-\phi')}$$

The rest of algebra is all fixed by closure

Check-point : $W(3)$

- $\Delta = X_{22} - \frac{2\pi}{k}\left(L X_{11} + 12 M X_{1111}\right)$

$$\Lambda = a X_{11} + b X_{12} + \varepsilon X_{22} + \eta X_{2222}$$
$$+ m X_{111} + n X_{1112} + p X_{1122} + q X_{1222}$$

field-dependent, source of nonlinearity

- equating each sides of

$$\Delta + \delta\Delta = \partial_t \Lambda + [\Delta, \Lambda],$$

$$b = \frac{1}{2}\varepsilon', \quad a = \frac{1}{2}\varepsilon'' - \frac{2\pi}{k}\varepsilon \cdot L - 2\cdot\frac{2\pi}{k}\eta \cdot M$$

.....

$$\delta L = -\frac{k}{4\pi}\varepsilon''' + (L\varepsilon)' + (\varepsilon' L)$$
$$+ 2(\eta M)' + (\eta' M)$$

$$\delta M = \frac{1}{288}\cdot\frac{k}{2\pi}\eta'''' + \cdots$$

$\rightarrow$ $\{L(\phi), L(\phi')\} = -\frac{k}{4\pi}\delta''' + (L(\phi) + L(\phi'))\,\delta'$

$$\{L(\phi), M(\phi')\} = (M(\phi) + 2M(\phi'))\,\delta'$$

$$\{M(\phi), M(\phi')\} = \frac{1}{288}\frac{k}{2\pi}\delta^{(5)} - \frac{5}{144}(L + L)\,\delta^{(3)}$$
$$+ \frac{1}{48}(L'' + L'')\,\delta' + \frac{1}{9}\frac{2\pi}{k}(L^2 + L^2)\,\delta$$

Fourier-mode decomposing

$$[L_m, L_n] = (n-m) L_{m+n} + \frac{c}{12} m(m^2-1) \delta_{m+n}$$

$$[L_m, V_n] = (2m-n) V_{m+n}$$

$$[V_m, V_n] = \frac{c}{360} m(m^2-1)(m^2-4) \delta_{m+n}$$

$$+ \frac{16}{5c} (m-n) \Lambda_{m+n}$$

$$+ (m-n)\left[\frac{1}{15}(m+n+2)(m+n+3) - \frac{1}{6}(m+2)(n+2)\right] L_{m+n}$$

where

$$\Lambda_{m+n} = \sum_{n=-\infty}^{\infty} L_{m-n} \cdot L_n$$

... this is classical $(5c+22 \to 5c)$ limit of Zamolodchikov's W_3 algebra!

- Many features persist in higher-dimensional contexts

[cf. A. Jevicki's talk]

except concrete identification of $W(\infty)$ symmetry algebra

Discussions

- $W(\infty)$ as asymptotic symmetry of AdS_3 higher spin gauge theory

- susy extensions interesting & nontrivial

$sl(2) \subset osp(n,2) \subset shs(n,2;\mathbb{R})$

$sl(2) \subset hs(2,\mathbb{R})$

$shs(n,2;\mathbb{R})$

no $sl(3)$ closed subalgebra exists

- dual $CFT_2 \stackrel{\sim}{=} (BI-DS-P)$ system at weak coupling

$\Rightarrow$ finite truncation to minimal model also interesting [Gopalkumar Gaberdiel]

- understanding 3-pt function is feasible from HS viewpoint

- $c = 3L/2G_N$ is independent of 't Hooft coupling

This implies that HS global symm. enhancement does NOT require xtra d.o.f.

- HS black hole description of each microstates should exist

.... GR BH uniqueness theorem is not applicable to HS theory

- understanding transition from tensionless string to HS gauge theory imminent